University of Alberta

Relations among *I*, *Thou*, and *It* In an Elementary Mathematics Professional Development Setting

by

Gladys Sterenberg ©

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to Gary

with love

Abstract

Mathematics is often viewed as a set of invariant prepositions, theorems, and axioms. However, from a historical perspective, these mathematical objects and results have evolved and are contingent on cultural developments within society. Various images of mathematics are envisioned by teachers. Studies of teacher learning suggest that teachers tend to make sense of new ideas through existing views of the nature of mathematics: in this way, beliefs about the nature of mathematics impact pedagogy. Indeed, as mathematician René Thom has said, “All mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics.”

In this study, I investigate the experiences of four elementary teachers as they engaged in a professional development project concerned with designing a series of mathematics lessons where historical stories were used as instructional tools. My dissertation considers how images of the nature of mathematics can be altered through immersion in the uncertainty and indeterminacy of historical mathematical stories. I seek to challenge the predominant view many elementary teachers have of mathematics as isolated facts and procedures that must be memorized. I do this by means of creating with the teachers a parable that presents mathematics as a vibrant and creative activity.

Drawing on Martin Buber’s notions of *I-Thou* and *I-It* relations and on David Hawkins’ related triad of *I-Thou-It* relations, I primarily investigate relations between the teachers and the mathematics being studied. In addition, I consider my relation to the mathematics and relations among the teachers and myself.

My analysis examines the conversational discourses that seem to offer expanded possibilities and insights within our learning community. Working with three emerging metaphoric themes, and drawing on Jerome Bruner's distinction between *narrative* and *paradigmatic* modes of thinking, I make the following claims. (1) Participation in this project fostered alternative images of mathematics as teachers grappled with philosophical tensions in mathematics. (2) Engagement with the history of mathematics by itself was insufficient to move teachers toward greater involvement with the paradigmatic dimensions of mathematics. (3) Nevertheless, teacher learning about mathematics occurred in this professional development setting because relations among the teacher educator, the teachers, and mathematics were robust.

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Prologue – Once Upon a Time[1]

Disconnection

> *I do not want to be here. As I sit passively in front of the computer terminal, my gaze sweeps around the computer lab, taking in images of other teachers dotting the perimeter. All of us have our backs to the centre of the room. We are isolated from one another. I glance at the principal who is standing by the door with his arms folded across his chest. It looks like he is blocking our exit. I frown as I quickly look back to the screen. I am here only because he requires us to attend this inservice workshop. A now-disembodied voice directs me to click on the mouse and drag the line to the centre of the page. I reach for the mouse, promptly execute this step, and wait for the next instruction.*

It is September 24, 1998. At this point, I have taught in schools for fourteen years and have benefited from numerous professional development experiences. Why am I finding this one so frustrating? Memories of other experiences of professional development surface. One memory I have is from November 18, 1995. I was seated at a table with four other teachers. The presenter from the curriculum division of the government distributed packages of worksheets containing cartooned word problems for mathematics. This experience became worthwhile because I was provided with one immediately useful teaching tool. Like many teachers, I learned by experimenting with the materials, in this case the worksheets provided by this 'expert', as I offered learning tasks to the students in my classroom. I appreciated being provided with a concrete, practical task that I could use in my classroom.

The voice interrupts my reminiscing. As I work through the next set of directions, I wonder if I am annoyed because the task we are working through has no immediate application in my classroom. Does this voice come from an expert who understands what it means to teach? I listen carefully. This voice is that of a consultant hired to provide inservice training. I turn to look at him across the classroom expanse. I have attended many inservice presentations provided by university consultants, district administrators, and guest lecturers. All of these have involved someone framed as an expert delivering a predetermined body of knowledge in a top-down fashion in what could be described as a traditional approach to training teachers. This format is familiar to me. I do not feel anger towards this particular consultant. My expectations for learning are not high, but I still may be provided with a teaching strategy I can use in my classroom. I respond to this traditional way of teaching by passively listening to his monologue and following his directions. Yet I still feel uneasy. I mechanically follow the next instructions. I know this content. I could teach this session. Could this be the reason for my resistance?

---

[1] This story takes place in 1998. I had reduced my teaching load to 50% as I began my studies as a full-time graduate student in a master's of education program. During that year, I wrote reflections on my teaching experiences in relation to the theories I was encountering. This story draws from those reflective writings. Much of my resistance to this particular professional development project arose from a dissonance between what I was learning about professional development processes and what I was experiencing. I responded strongly to being treated in what I perceived to be an aloof manner. As the year progressed, I decided to consider other career alternatives. 1998-1999 became my final year of teaching elementary school.

I recall another memory from August 27, 1992. I was sitting in a row of chairs facing the district curriculum coordinator who was delivering a series of workshops on language learning. Having transferred from a junior high mathematics classroom, I was unfamiliar with how children learned to read, write, and communicate through speech. I was eager to learn, but this expert was intent on telling us about the new resources being developed by people in his office. A teacher interrupted him by asking about the agenda for our meeting. He ignored her question and continued. This seemed arrogant and condescending. I panicked about teaching language arts in the upcoming school year.

This memory fades as I try to reconnect with this consultant's agenda. The content of his presentation does not capture my attention. It is not "differentiated to meet my learning needs." Perhaps this contributes to my negative response. I do not feel that I am learning what he intends for me, yet I have engaged myself in thoughtful reflection, so the time has not been completely wasted. Maybe time is the issue. As I glance at the clock above the door, I notice the principal. My teeth clench again as I remember the last staff meeting.

My memory of the staff meeting on September 10, 1998 is very clear. We were discussing the agenda item about professional growth plans. I listened to the principal describe how several teachers had indicated an interest in learning how to use a computer. I tuned out the rest of his comments as I thought about what I wanted to focus on. Suddenly it was quiet. Everyone was looking at me. I hastened to apologize as the principal repeated his question: What does the research show about sustained professional development? I responded that often it was more effective than a single workshop. He smiled and then declared that all staff members would participate in a nine-month series of computer workshops to be held after school on alternate Wednesdays. I blurted out that he could not be serious and immediately felt embarrassed. I had been disrespectful. He angrily stated that there would be no exceptions.

I am jolted back to the computer lab by a movement of his head. As I make eye contact with him, I realize that I am resentful because he has not respected my professional knowledge. This inservice workshop is mandatory, decreed from above, one size fits all. I do not want to be here. It is not part of *my* professional growth plan. I experience the tensions between my desire for autonomy and his desire to guide my development. I need the time to prepare for tomorrow's lessons before I begin my evening class. Our relationship is interrupted by this imposition. He has determined what I should learn. I am voiceless. How can I terminate this experience?

I resist his authority and this affects my learning. My relationship to the content is also compromised because I am not able to see the possibilities for learning. Am I resistant to top-down, authoritative 'experts' delivering a preconceived body of knowledge? Am I resistant to learning from people who do not appear to respect the experience of teachers? How can hierarchical structures of professional development be reframed? Do professional development experiences have to be like this? I turn my back on him as I recall another memory.

Synergy

I laugh as I lean forward to hear my colleague's next words. In jagged text, I jot down our brainstormed ideas as they are blurted out. I struggle to keep up. What are other possibilities for teaching geometry and *measurement? I listen intently to a second colleague who begins listing connections between mathematics and building a house. Suddenly there is silence as we look at each other and nod. Quiet laughter follows. We scribble as we translate our ideas into student tasks. I glance at my watch, surprised to find that we have been working for more than five hours. This is synergy. But where is our curriculum leader?*

It was October 1, 1993. Working under the guidance of a lead curriculum teacher, twelve of us gathered to understand the new mathematics curriculum. We talked about constructivism and social constructivism. We brainstormed and created lists that described our ideas of the nature of mathematics, of learning and teaching, and of children. She listened to our ideas, wrote them on chart paper, and asked probing questions that challenged our way of thinking. We used our ideas to create student tasks that addressed the learning objectives of the new mathematics curriculum. Since textbook materials were not yet available, the focus of our project oscillated between developing student tasks and interpreting the underlying theories of the new curriculum. We began by brainstorming themes for study. I felt comfortable and willingly participated by making several suggestions. Again, the curriculum leader recorded these on chart paper. Several themes for study emerged from our group conversations. We formed smaller groups and chose an area of interest to focus on. At this point, she withdrew from our group to circulate among the other groups.

During our lunch break, I tentatively asked her about her approach to professional development. I viewed her as an expert. She leaned forward as she told me that she wanted to provide experiences for teachers that reflected her evolving notions of pedagogy. She described a university-level course that she was taking on constructivist theories of learning and explained that she was trying to be a facilitator. For the remainder of the meal, we talked about co-construction and learning. To what extent was the co-construction of knowledge dependent on a leader? How could the distance between the expert and the teachers be negotiated? How could all members participate in learning within a community?

After lunch, I worked with two other teachers to create an integrated curriculum and encountered an extremely creative process of learning. It was intense, exciting, challenging, and enjoyable. As a smaller group of teachers, we chose to work on a house construction project that was of personal interest to us. We were provided with release time to do this and we made plans to pilot the tasks in our classrooms. It was our choice to participate in this project and all of us were experienced teachers interested in mathematics. In fact, many of us were lead mathematics teachers in our schools. We understood mathematics in a deep way and our content knowledge was strong. Perhaps this contributed to the joy we encountered as we learned.

This encounter of learning was qualitatively and profoundly different from any of my previous experiences. Our curriculum leader assumed that we would produce a teaching unit based on the new curriculum. To some extent there were preconceived outcomes and she provided necessary direction towards these goals. She had an implicit agenda for teachers to embrace emerging ideas of mathematics learning and implement a new curriculum. Yet this encounter seemed to expand the possibilities for my professional learning. We were constrained by her expectations of a product correlated to some aspect of the prescribed mathematics curriculum, but we were able to craft this teaching unit in a creative way within these constraints.

During this process of learning, my own perspective of mathematics changed dramatically. Rather than viewing mathematics as static knowledge, I contemplated this notion of constructivism. I began to think about mathematics as a human activity with historical and social origins. A reorientation of my thinking about mathematics required a shift in my understanding of the human role in mathematical development. I was resistant. For many years, I had understood mathematics as a set of infallible rules devoid of context. This absolutist approach reinforced the notion that knowledge was certain and unchangeable and that mathematical knowledge was based on reason alone. I began to understand how mathematical knowing could occur in a community of learners and how learning mathematics could be viewed as a process as well as a product. Indeed, I encountered a shift in my beliefs about the nature of mathematics. I was learning.

What were the interactions between views of the nature of mathematics and possibilities for learning within inservice settings? How were my views of mathematics linked to my experience within this approach to professional development?

Again my thoughts are interrupted by the consultant's voice. He is directing us to watch what he is constructing on the board. I try to attend. My gaze sweeps the room again. Some teachers are doodling, one teacher is marking student work, several teachers are working on their computers, a few teachers are watching him. Many of us do not seem to be engaged in this experience. I glance at the principal. He yawns and shrugs at me. I look back to the computer screen as I recollect my own attempt to lead teachers.

Ambivalence

> *"We cannot throw the baby out with the bath water!" She slams down the new textbook and crosses her arms across her chest. As I sit back, I prepare to ignore yet another tirade from this 'old school' teacher. Mentally planning tomorrow's lesson, an emphatic comment filters through my barricade against this onslaught. "There is nothing wrong with the way I teach mathematics." Startled, I lean forward, maintaining eye contact and listening attentively. Tears fill her eyes and I breathe deeply with heaviness. When did I become the expert authority, solely concerned with imposing my agenda on these learners?*

It was March 19, 1997. I was a teacher and a curriculum leader; I was framed as the expert this time as I worked with the intermediate teachers at my school. I was trying to replicate my previous synergetic encounter of working on this new curriculum. One teacher said she hated mathematics, she did not understand it, and she did not enjoy teaching it. Another agreed with her. I was surprised by the intensity of their reactions, as many of them were experienced teachers and had mentored me throughout my teaching career in this school. I was trying to participate as a colleague and teacher during this meeting, but I had been appointed chair by the principal. My intention was to explore the new curriculum, but his agenda was for us to make recommendations to the school board regarding the purchase of new mathematics textbooks. We had a mandate and a leader. I struggled with the changing dynamics among us. My relationships with the other teachers had been quite strong in previous years. I was a colleague and, in many cases, a friend. But our interactions changed from a focus on understanding the new curriculum to one of choosing a new textbook. Most teachers felt that this decision would affect their teaching and for the first time, the curriculum changes became tangible.

We began by investigating *Interactions* (Hope & Small, 1997a; Hope & Small, 1997b; Hope, Small, & Drost, 1997), at that time the only textbook series authorized by the provincial government. This particular textbook series represented a dramatic movement away from traditional approaches to teaching and learning mathematics. I loved it. I was able to use the text materials to develop projects and collaborative learning tasks to support my evolving beliefs about the nature of mathematics. However, these text materials did not include many drill and practice exercises. Tasks involving independent work were not included and formal tests were not supplied. The teachers in the group resisted changing from the familiar lesson model of 'exposition – examples – exercises'. These teachers struggled with content knowledge and their own confidence to teach mathematics. Although we had shared our struggles and triumphs, they were resistant to embrace changes when they were faced with the textbook embodiment of these ideas.

I recommended the text materials. The other teachers did not. One teacher did not want to throw the baby out with the bath water; another teacher was adamant that she would not follow yet another trend. We decided to compromise by correlating the new curriculum to both the old and the new textbooks. I was not happy with this outcome, because I believed that change would not occur in the teaching practices of the others. I had become focused on reforming others and on imposing my agenda for change. Hence our relationship became forced. This was not a rich encounter for me. Why had we not reached a consensus about the nature of mathematics and the new curriculum? Why were there vast gulfs between our individual notions of how mathematics should be taught? Why were our conversations so disorderly and confusing? How did our individual experiences influence our interpretations of the nature of mathematics? To what extent did the content of the professional development encounter impact our learning?

We were all held captive by the principal's mandate to recommend a textbook. There was no sense of community as we struggled with the imposition of enacting the new mathematics curriculum. This was not a challenge with which we had chosen to engage; rather, we were appointed members of an externally imposed committee. I had no autonomy and my professional learning was limited.

The image of a blinking light on the computer screen reappears. I glance at the clock; I have endured this session for too long. I am uninterested in the content of this workshop. I have a disconnected relationship with the principal and the consultant. I take a deep breath, log off, and stride purposefully to the door. The principal asks, "Where are you going?" I mumble that I have another class to attend and keep walking, silently shaking.

Chapter 1 – Situating the Study

Experiencing Mathematics

> *The students sat still in stunned silence as the words of one of them reverberated through the room: "I hate this! I'm so stupid! I don't have to take this course! You are a terrible instructor!" I ducked an onslaught of flying pencils and books. I had just started teaching the third class of an undergraduate course on mathematics curriculum and instruction by presenting a problem for the students to work on. I felt sick and announced a break. As I approached the young woman who had made this outburst, I was hesitant. This grade 3 mathematics problem should not have been so difficult and I was completely taken aback by the intensity of her reaction. "Perhaps you should drop the course." I offered this cautiously and was immediately ashamed. She defiantly responded, "No. I'm not stupid in my English classes. In fact, I'm the top student." I offered her a tissue as I tried to sort out this discrepancy. I decided to continue this class by talking about mathematics as a language. The students remained quiet.*
>
> *(Personal reflection, July, 2001)*

Studies of teacher learning suggest that prior experiences of learning mathematics are intertwined with teachers' views of the nature of mathematics. These images of mathematics influence pedagogy and can limit the implementation of mathematical reforms, even when there is a personal desire to change (Stipek, Givvin, Salmon, & MacGyvers, 2001). Teachers tend to make sense of new ideas through existing views of the nature of mathematics. Because these new ideas may not resonate with teachers' own experiences of learning mathematics, it is possible that the effect of reforms is limited. In this way, images about the nature of mathematics impact pedagogy. Indeed, "all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (Thom, 1973, p. 204).

The preservice teacher described above seemed unable to view mathematics in a problem-solving context. She believed that mathematics was solely arithmetic and that teaching mathematics involved the development of computational skills. Mathematical knowledge was perceived by her as certain, unchangeable, and absolute. Her previous experiences of mathematics had focused on training the mind through direct instruction of algorithms and mathematical procedures. During the course, many of my attempts to engage preservice teachers in mathematical tasks reflecting constructivist learning theories[2] were resisted. Some students seemed unwilling to relinquish their absolutist views of mathematics, even though they had described their previous learning experiences of mathematics in negative terms. I experienced many challenges when designing and implementing a course prompting preservice teachers to consider alternative notions of mathematics teaching and learning in an era of reform.

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[2] von Glasersfeld (1991), in his theory of radical constructivism, draws attention to the notion that mathematical knowledge is actively built up by the individual in her or his mind. Constructivists focus on mathematical understanding, not training. Social constructivist theories suggest that mathematical learning involves enculturation into socially established mathematical practices. Many of these interactive learning theories are derived from the work of Vygotsky (1978), who proposes that complex mental functions develop in historical and cultural contexts and that learning is both an individual and social process. Cobb, Jaworski, and Presmeg (1996) present the view that mathematical knowing and doing are social and cultural activities. Although constructivist and social constructivist theories of learning emerged during the

Many of the challenges I faced in this situation were similar to those encountered by teacher educators in professional development settings. Based on their research of teacher learning and professional development, Stein, Smith, and Silver (1999) suggest:

> Meeting those [reform] goals and standards will require a great deal of learning on the part of practicing teachers, the vast majority of whom were taught and learned to teach under a different paradigm of instruction and learning. The kind of learning that will be required has been described as transformative, that is, as requiring wholesale changes in deeply held beliefs, knowledge, and habits of practice. (p. 238)

Experiences of learning influence teachers' pedagogy. Even though most elementary teachers do not want their students to have negative mathematical experiences, research indicates that teachers tend to teach as they were taught (Ball, 1990). Furthermore, teachers have unintentionally participated as students in an apprenticeship of observation (Lortie, 2002) and may experience resistance when challenged with implementing mathematical reforms.

In addition, many elementary teachers in North America lack a profound understanding of fundamental mathematics and this limits the impact of new ways of teaching (Ball & Cohen, 1999). Schifter (1998) describes a teacher who studied reform documents, critically reflected on constructivist views of learning, and attended a variety of workshops and courses. This teacher portrayed her attempt to develop a practice that built on her students' ideas as difficult and unsatisfying. She identified her lack of knowledge about mathematics as a limitation. Schifter asks, "What is a teacher, who is working to establish a classroom culture in which intellectual authority is shared, to do when students propose ideas that are surprising, incorrect, or even, it may seem, incomprehensible?" (p. 56). It appears that a flexible knowledge of mathematics is needed. Ma (1999) suggests that "the mathematical understanding of a teacher with a procedural perspective is fragmentary" (p. 119) and that elementary mathematics is made coherent by emphasizing connections among algorithms, and between algorithms and other topics in mathematics. It would seem, at least for some elementary teachers, that experiences of learning algorithms in isolation contributes to negative experiences of mathematics and a weak understanding of mathematics.

Such research suggests that background experiences as learners of mathematics, views about the nature of mathematics, and content knowledge of mathematics are all related to pedagogy. Understanding new images of mathematics is very challenging and can contribute to teacher resistance. What are the possibilities for experiencing mathematics in a positive way? How can alternative experiences be offered to elementary mathematics teachers in professional development settings?

---

second half of the twentieth century, they did not impact mathematics curricula in this province until the 1990s.

Relating to Mathematics

Davis and Hersh (1981) describe a variety of mathematical experiences and conclude:

> [The meaning of mathematics] is to be found in the shared understanding of human beings, not in an external nonhuman reality. In this respect, mathematics is similar to an ideology, a religion, or an art form; it deals with human meanings, and is intelligible only within the context of culture. In other words, mathematics is a humanistic study. It is one of the humanities. (p. 410)

This perspective differs greatly from traditional views that informed many of my experiences of mathematics. A reorientation of my thinking about mathematics required a shift in my understanding of the human role in mathematical development. For many elementary teachers, mathematics is understood as a set of infallible rules devoid of context. This absolutist approach reinforces the notion that mathematical knowledge is certain and unchangeable and is based on reason alone. Observations of the world and our experiences in it are ignored. Lakatos (1976) explains:

> In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as an ever-increasing set of eternal, immutable truths. Counterexamples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with disguised monster-barring and proof-generated definitions and with the fully-fledged theorem, and by suppressing the primitive conjecture, the refutations, and the criticism of the proof. Deductivist style hides the struggle, hides the adventure. The whole story vanishes, the successive tentative formulations of the theorem in the course of the proof-procedure are doomed to oblivion while the end result is exalted into sacred infallibility. (p. 142)

During the implementation of the new elementary mathematics curriculum in 1990s, I was responsible for providing inservice opportunities for my colleagues at our school. A memory of *Ambivalence* described in the Prologue (see p. 4) portrayed the hostility, frustration, and stubbornness teachers can experience when faced with new ways of teaching and learning mathematics. Many of them stated emphatically that they hated mathematics. While teaching prospective elementary mathematics teachers, I came across similar reactions. My recollection of *Experiencing Mathematics* (see p. 6) described one such situation. In order to diffuse the apparent hostility, I decided to talk about mathematics as a language and introduced tasks based on literature. I saw students begin to take risks. During one class, the student I described volunteered to read a story she had found that showed mathematical patterns.

On the course evaluation, many students wrote that they struggled with mathematics and appreciated my approach to teaching it. For many of them, this was the first time they had encountered an alternative view of mathematics. By engaging in conversations about mathematics as a language, some students were able to view mathematics as another way of talking about the world we live in. We investigated the nature of mathematics as dependent on cultural understandings and changing over time. We talked about mathematics as an invention by people. Many students experienced mathematics in a manner that embraced contextual, relational, and storied ways of knowing.

Contextual and relational perspectives support the view that experience is storied. Davis (1996) writes, "Enactivist theorists seek to reawaken our awareness of the active, negotiated, storied, and relational natures—that is, to give a dialogical rather than a monological account—of our selves" (p. 173). Researchers who suggest human experience is narrative inform my view of mathematics as a humanity. Indeed, the metaphor *mathematics is story* is connected to my understanding of mathematical experiences. Borasi and Brown (1985) suggest that a storied approach to learning mathematics promotes natural and spontaneous mathematical learning. Burton (1996) writes:

> I claim that a narrative approach to mathematics and its pedagogy is consistent with a view of mathematics as being socially derived and with the understanding of mathematics as being socially negotiable. [...] By engaging with the narrative, we place the mathematics in its context and personalize it, making it come alive to the conditions of the time. Context provides meaning. (pp. 32-33)

The metaphor *mathematics is story* embodies the characters, settings, plots, and problems of life. Although somewhat related to word problems, the intentions of stories are to show our relationship to our world. They are meant to recall our lived lives and to prompt thoughtful consideration; they are not intended to be discarded as exercises or, as Gerofsky (2004) terms it, "as throwaways" (p. 41). Stories in this narrative sense become dramatizations of mathematics and support rich, emotional, and engaging experiences of mathematics.

Derived from the Latin word *historia,* the terms *story* and *history* share a common background. A story can be described as a historical relation or a life-giving form of history. Early meanings of history included notions of narrative tales of past events, either true or imaginary (*OED*, 1989). By considering relations among story, history, and life experiences, I suggest that the study of the history of mathematics can reorient and reconnect mathematics to its human context.

The study of the history of mathematics supports the view of mathematics as a humanity. Lakoff and Núñez (2000) present mathematics as a "magnificent example of the beauty, richness, complexity, diversity, and importance of human ideas. It is a marvelous testament to what the ordinary embodied human mind is capable of—when multiplied by the creative efforts of millions over millennia" (p. 379). When storied aspects of mathematics are returned to the classroom, mathematics becomes interesting and creative. Davis (1996) argues that mathematics becomes an inhumanity when we ignore human experiences: "It is thus that we have created a system that values compliance over creativity, that spawns destructive behavior by destroying our experience, and that conditions learners to reach for the formulaic ahead of the imaginative" (p. 281). How can storied ways of knowing mathematics be enacted in professional development settings?

Studying the History of Mathematics

The grade 5 students listened intently as I told the classic story of Archimedes and the golden crown.[3] *I presented the problem faced by Archimedes and the students*

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[3] I crafted the story from biographical notes presented in the sidebar of a teacher's guide. In this story, King Hieron is concerned that the crown he has commissioned is not pure gold. He wonders if the goldsmith has

rushed to form groups as they brainstormed possible solutions. Their interactions were focused on the context of the story. After a few minutes, I offered more details of the story to the class and together we worked on possibilities for investigating this problem. Finally, I told the rest of the story. Eureka and spontaneous laughter! The students talked excitedly as we went to the science lab to test Archimedes' notions. This was a rich learning and teaching experience. Why don't I tell stories more often in mathematics class?

(Personal reflection, March, 1999)

What are the possibilities for learning mathematics within a historical context? To what extent can the study of the history of mathematics prompt elementary teachers to consider the human agency in the discipline? As Ernest (1991), Fauvel (2000), and Lakatos (1978) suggest, can this provide a means for altering mathematical experiences? What are possibilities for incorporating stories into the study the history of mathematics?

Many teacher education programs for secondary mathematics teachers include courses on mathematics history. While several studies have explored the effect of using historical materials with preservice teachers (e.g. Furinghetti, 2000; Krussel, 2000) and students (van Maanen, 1991), they have tended to focus on secondary school mathematics where a considerably stronger background in mathematics was assumed.

Recent studies are beginning to investigate the impact of learning about the history of mathematics on elementary preservice teachers. For instance, Stander (1991) found that studying the history of mathematics did not significantly improve the attitudes of primary preservice teachers toward mathematics. Two experiments were conducted over a short period of time and historical topics were used as enrichment materials. Because the historical topics were not integrated into the mathematics course, preservice teachers may have been reluctant to devote extra time to the investigation of these.

In another research project, Philippou and Christou (1998) found that preservice primary teachers who participated in a mathematics preparatory program based on the history of mathematics significantly improved their attitudes toward mathematics. This was noteworthy, as a high proportion of preservice teachers entering the program had very negative attitudes. In addition, many of these preservice teachers reported increased perceptions of the usefulness of mathematics in society and viewed themselves as interested in, motivated by, and capable of doing mathematics. One participant described the experience:

> History of mathematics provided me with a variety of interesting, new, experiences…Through the journey I realized that mathematics has always been and continues to be a very useful subject…I appreciated the efforts of people to use mathematics to solve daily problems. The course showed me that mathematics is, at least sometimes, a human activity. I felt more confident when I realized that even great mathematicians did mistakes as I frequently do. (p. 202)

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substituted silver inside the crown in order to keep the valuable gold for himself. While pondering this problem, Archimedes decides to visit the public bathhouse. He notices that the water level rises when he enters the tub and connects this law of buoyancy to the King's problem. In his excitement, he jumps out of the tub and runs down the street shouting "Eureka, eureka!" A vivid and detailed version of this story is presented in Reimer and Reimer (1990).

The program consisted of two mathematics content courses based on the history of mathematics and one methods course. It was designed and implemented over a three-year time period. Topics reflected the historical development of basic concepts. Included in the program were number systems, the Pythagorean theorem, Euclid, Archimedes, hyperbolic geometry, matrix algebra, sets and equivalence, logic truth tables, and Boolean logic. Two assumptions made when designing the program were that "most of primary mathematics has its roots in the early historic period, and that preservice teachers would be motivated to read mathematics in the context of their initial development" (p. 193).

The authors of this study noted that factors other than the study of the history of mathematics, such as the emphasis placed by instructors on mental models, may have contributed to attitudinal changes. Also, the long-term effect of the program on both attitudes toward mathematics and pedagogy was not addressed. Despite these limitations, the study by Philippou and Christou contributed to my research design in two ways: (1) I was aware that the study of the history of mathematics would involve an investment of time on the part of teachers, and (2) the topic of number systems offered a possible starting point for this investigation.

Experiences of six elementary teachers engaged in incorporating historical and multicultural material into their teaching were investigated by Percival (2004). In her doctoral study, teachers had a limited knowledge of cultural mathematics and time constraints limited the exploration of such ideas. Although teachers seemed to recognize the benefits of promoting the cultural background of mathematics, Percival suggested that only those who had a particular interest in this area would incorporate it into their teaching. For me, this work reinforced the importance of being sensitive to issues of teachers' time commitments and issues of engaging in the task of studying the history of mathematics. In order to ease teachers into, and prompt their engagement with, the investigation of historical contexts in mathematics, I decided to incorporate the writing of historical stories into my study. I was concerned that the mere study of historical information might simply feed into teachers' negative experiences with mathematics.

Historical information is available to elementary teachers. Typically, biographical facts are written in sidebars of textbook margins and contain sparse details. The connection of the person to the mathematics is weak. For example, in *Quest 2000: Exploring mathematics, grade 4* (Wortzman et al., 1996), the student textbook used by many teachers in Alberta, four references to the history of mathematics are included. These are set apart from the labelled lessons as studies of people, society, and mathematics. Many of these contain vague connections where the content is loosely linked with the task presented. An example is the presentation of instructions for making a paper airplane: "As far back as 4000 years ago, hunters used flint arrowheads in the shape of isosceles triangles" (p. 64). Another example of different procedures for multiplication refers to the lattice methods used in India and the doubles methods used by Egyptians. A picture of Egyptian hieroglyphics accompanies the statement, but it is not clear how this relates to the doubling procedure of multiplication. The authors have not included reasons for these inventions and have neglected to situate the inventions within a historical context.

The teacher's guide includes a note that these pages can be used to expand students' understanding of the use of mathematics in historical and cultural situations.

However, this is written in a sidebar and hence serves to marginalize the text it contains. Additional information about the historical context is not provided for the teacher. These pages are not included in the unit lessons and therefore can be viewed as enrichment tasks that are extra to the program. I would expect that these would be treated as extra material and teachers would have similar responses as the preservice teachers in Stander's (1991) study. When mathematics is presented in this manner, the historical context and the social implications of the inventions are lost. This approach to the study of history contributes to the notion of mathematics as a collection of disembodied, disjointed facts.

Professional resources about the history of mathematics exist. However, these tend to address a specialist interest and are often aimed at the secondary or post-secondary level. In an overview of historical resources for teachers, Rogers (1991) organizes references into categories that include encyclopaedias and dictionaries, chronological tables and charts, bibliographies, source books, and general histories of mathematics. Expertise in mathematics is assumed. No book on his list focuses on the stories of mathematics.

Because of limited mathematical backgrounds, many elementary teachers tend to rely on textbook-centred instruction (Ball, 1990). However, since the implementation of the *Curriculum and evaluation standards for school mathematics* (NCTM, 1989), alternatives to textbook-centred approaches are emerging. Collections of mathematical stories are becoming more available as the current interest in mathematics and literature expands (e.g., Haven, 1998; Reimer & Reimer, 1990). Internet resources for incorporating historical topics into elementary mathematics classrooms are increasing. This suggests that elementary teachers are interested in the history of mathematics.

How can the study of the history of mathematics be prompted within a professional development setting? What might be its impact on teachers' experiences of mathematics and their views of the nature of mathematics? Will this expand the possibilities of knowing mathematics for elementary teachers?

Relating Through Professional Development

My experiences in a variety of professional development settings have informed my perspective of learning and teaching mathematics within a context of relations. Like Darling-Hammond and Sykes (1999), I believe that the professional development of teachers is essential in improving mathematics instruction in schools. Certainly, my own professional learning experiences provoked me to view mathematics as a humanity.

How have my experiences of professional development influenced my study? When I return to the memories in the Prologue, it is evident to me that I am oriented to and motivated by relationships. Relationships described in terms of disconnection, synergy, and ambivalence have impacted my learning. Many of these involved distance, as externally mandated agendas and the transmission of specific teaching techniques limited my opportunities for collegial interactions. Often the teacher educator did not participate as a learner in the setting, but instead acted as a facilitator. Sometimes relations between the leader and myself were hampered, as revealed in my recollection of *Disconnection* (see p. 1).

However, there was a qualitative distinctiveness in my experience described in my memory of *Synergy* (see p. 2). In this setting, I experienced a synergetic relationship both with my colleagues and with the mathematics we were working on. I described this synergy as offering expanded possibilities for learning through our combined interactions. In other words, our combined interactions provoked learning that would not have been possible as individuals. The sum was greater than the parts. This experience prompted me to think about professional development as a participatory endeavour within a community.

In many contexts, my relationship to the mathematics was limited, because it was mediated by an expert. I did not engage directly with the mathematics, but listened to disconnected, authoritative explanations of it. In this study, I offered an alternative to viewing mathematics as arithmetic procedures by engaging elementary teachers in the study of the history of mathematics. For those teachers in my study who had had previous negative experiences in mathematics, and for those who tended to focus their teaching on the direct instruction of algorithms, I prompted reflections on views of mathematics as a humanity. This perspective of mathematics was to prove very challenging, but story and history allowed teachers to engage in mathematics in a comfortable and familiar setting. I believe this lowered their resistance and contributed to more positive attitudes toward mathematics.

In this research study, I worked with four elementary teachers to investigate how mathematical experiences and images of the nature of mathematics could be altered through immersion in the uncertainty and indeterminacy of historical stories in mathematics. My dissertation considered teachers' experiences of writing historical stories of mathematics and designing curriculum tasks. By creating a curriculum that explored mathematics as a vibrant and creative activity with them, I offered a professional development opportunity that expanded possibilities for creating new stories of mathematical experiences.

Drawing on Buber's (1923/1970) notions of the relations of *I-Thou* and *I-It* and Hawkins' (1974) triad of the relations of *I-Thou-It*, I investigated relations between the teachers and mathematics, between myself and the mathematics, and among the teachers and myself. I began with a Prologue that introduced the background of my study within narrative accounts of my own experiences of professional development. In this chapter, I have offered reasons for my interest in the research and introduced the metaphor of *mathematics is story*. Chapter 2 presents my understanding of possible relations in professional development settings, both for individual learning and within learning communities. I provide a description of the methodology of my study in Chapter 3. Chapters 4 – 8 investigate relations between the teachers and the mathematics being investigated. I explicate metaphors explored by the participants and analyze the metaphor *mathematics is story*. Chapter 9 describes relationships among members of the learning community within this particular professional development setting. In this concluding chapter, I reframe my inquiry and present some implications for teacher education programs for preservice and inservice elementary teachers of mathematics.

Three interludes are interspersed throughout the writing. These are personal reflections of my evolving relationships to the mathematics and to the teachers as a researcher and professional development educator. The epilogue offers an opportunity for me to revisit the beginnings of this research and to contemplate my contribution to theories of professional learning.

Beginning Another Story

Once upon a time, my experiences of and encounters with professional development ranged from disconnection to synergy. By participating in a variety of approaches to professional development, I encountered many learning opportunities as a teacher. Like Borasi and Fonzi (2002), I have come to believe that traditional inservice workshops are limited in fostering communities engaged in mathematics learning. What has become increasingly important to me is the notion of community in professional learning. These relations are what seem to offer rich possibilities for professional development.

In my study, I investigated one possibility for professional mathematics learning. Engaged in a community of learners with four teachers, we created and implemented a series of lessons based on mathematical story. This provided a beginning space for both the teachers and myself to explore our relationships both to mathematics as a subject and to one another. For me, this community of learners challenged my complacent notions of the ease of co-construction and I encountered tensions and joys in relation to these teachers, as I sought to become a full participant in this study.

Three research questions framed the beginning of this study: (1) What are teachers' encounters with mathematics when creating and implementing a series of lessons based on a historical story for elementary mathematics classrooms? (2) What are my experiences of working on the mathematics as a teacher educator? (3) How do we, as members of a learning community, experience our relationships with one another? As the study progressed, further research questions, that focused on teachers' images of the nature of mathematics, emerged: (1) How did engagement with the history of mathematics impact images of mathematics? (2) What images of mathematics were envisioned by the teachers in the study? (3) How did relationships among the teacher educator, teachers, and mathematics influence teacher learning about the nature of mathematics?

Many researchers in mathematics education have presented novel approaches for professional development. In order to situate my research within that of others, I consider the experiences of researchers engaged in systemized professional learning. In the next chapter, I present my current understanding of the interplay between these innovations and a theoretical framework of relations.

Chapter 2 – Considering Relations as More Than an Affair of Mirrors

Mutual Learning of Mathematics

> *I was reading* Chaos *(Gleick, 1987) and was amazed by the images: The Lorenz attractor, the Koch curve, the Mandelbrot set, and fractal clusters. Drawn into the stories told in this book, I was fascinated by these new mathematical possibilities for describing our world. Nonlinearity, pluralism, and the dynamic nature of chaos resonated with me. Yet it was the computer-generated pictures that held my attention. I was enthralled by the colour and fluidity of shape. I began to play with the mathematics. How were these patterns created? I suddenly stopped my calculations and jerked back in my chair. What if weather patterns, population patterns, and economic patterns were chaotic? I imagined tree branches, coastlines, and blood vessels as fractals and was fascinated by the harmonious arrangement of order and disorder occurring in natural objects. This mathematics was beautiful.*
>
> *(Personal reflection, February, 1999)*

This encounter with mathematics was part of another professional development endeavour. I was enrolled in the second semester of my master's studies in the area of computer technology and was engaged by this emerging field. Like Capra (1996), I was attending to the relationships and patterns of this new mathematics:

> It is qualitative rather than quantitative and thus embodies the shift of emphasis that is characteristic of systems thinking—from objects to relationships, from quantity to quality, from substance to pattern. [...Mathematicians] have discovered new qualitative patterns of behaviour of those complex systems, a new level of order underlying the seeming chaos. (p. 113).

I had become entrenched in my experiences as a mathematics teacher and divorced from my undergraduate experiences as a mathematician. However, I was once again relating to the beauty, elegance, and imagery of mathematics. In that jolting moment of clarity, I saw expanded possibilities of how I could relate to this new mathematics. Reasons for my reconnection to mathematics were similar to what Capra describes:

> The great fascination exerted by chaos theory and fractal geometry on people in all disciplines—from scientists to managers to artists—may indeed be a hopeful sign that the isolation of mathematics is ending. Today the new mathematics of complexity is making more and more people realize that mathematics is much more than dry formulas; that the understanding of pattern is crucial to understand the living world around us; and that all questions of pattern, order, and complexity are essentially mathematical. (pp. 152-153)

The professor of this course approached teaching in a different manner from many of the experts involved in inservice education. Every week, we met in a coffee shop to discuss our personal investigations in the area of technology and education. Our conversations were energizing, compelling me to investigate diverse areas of research such as chaos theory. For me, this was another encounter with synergetic relations.

I believe my relations with the mathematics and the relations among the members of this class contributed to the richness of this encounter. How did my interactions influence my learning as a teacher? To what extent did the quality of my relationships to the content and to the teacher educator affect my professional learning?

Alternative Views of Professional Development

Many of my experiences described in the Prologue and Chapter 1 exemplified the tensions of teacher autonomy and administrator authority. In this chapter, I begin with generalizations from investigations of alternative approaches to the traditional training model of professional learning. The general principles presented by researchers interested in such explorations promote strong relations between the teacher educator, the teacher, and mathematics. Guidelines arising from this literature informed the design of my study, which I discuss in Chapter 3.

During the past two decades, opportunities for mathematics teachers' professional development have focused on the implementation of the *Curriculum and evaluation standards for school mathematics* (NCTM, 1989). *The professional standards for teaching mathematics* (NCTM, 1991) was commissioned to promote this vision through the professional development of mathematics teachers. Six standards were presented: experiencing good mathematics teaching, knowing mathematics and school mathematics, knowing students as learners of mathematics, knowing mathematical pedagogy, developing as a teacher of mathematics, the teacher's role in professional development. While these standards were descriptions of what was desirable, they were not based on results of research studies.

Researchers began to examine those conditions of professional development that seemed to result in qualitative change in teaching and learning mathematics. Much of their work focused on descriptions of programs designed to promote these conditions. They suggested that learning occurs when teachers choose to be involved, when lived experiences are investigated, when teachers reflect on teaching practices, and when teachers participate in collaborative projects (Castle & Aichele, 1994). In addition to these conditions, other researchers suggested that teachers must become adaptive agents and have to improve their subject matter learning (Cooney, 1994; Hyde, Ormiston, & Hyde, 1994). Clarke (1994) summarized research for the professional development of mathematics teachers and presented ten key principles. These focused on encouraging teacher choice, working in groups, addressing impediments to teachers' growth, basing research in the classroom, soliciting commitment, allowing time for planning and reflection, regarding participants as partners, providing ongoing support, and encouraging participants to set further goals.

Despite discussions of principles and conditions of professional development found in the literature, little is known about how teachers learn. I am challenged by Ball's (1996) claim:

> The teaching we are trying to help teachers learn is underdetermined, not reducible to simple programs of practice. Likewise, our understanding of professional development that can support teacher learning is a mix of fairly solid ideas, beliefs, myths, and conjectures. (p. 507)

Her working ideas include a consideration of prior beliefs and experiences, subject-matter knowledge, knowing the students, practice-based contexts, time, reflection, follow-up, modeling, and teacher control. She does not identify these ideas as tenets of knowledge because she believes they are unevenly analyzed and warranted. In the decade since this article was written, what are the research findings on professional development? To what extent are robust theories of professional learning emerging?

As described in the Prologue and Chapter 1, resistance to change was linked to entrenched notions of the nature of mathematics, teachers' experiences, and a lack of mathematical knowledge. Recent professional development initiatives have begun to promote teacher inquiry in an effort to encourage pedagogical change. Research on teachers' mathematical and pedagogical knowledge (Ball & Cohen, 1999; Ma, 1999) and studies of teacher resistance (Fullan, 1991) have informed the shift from training to inquiry approaches. The underlying premise of much of this research is that school improvement relies on "the development of a highly qualified teacher workforce imbued with the knowledge, skills, and dispositions to encourage exceptional learning in all the nation's students" (Darling-Hammond & Sykes, 1999, p. xv). Professional development in this context is viewed as essential, not superfluous.

Although many generalizations for professional development approaches have been offered, the lack of research-based theories of teacher learning is evident (Ball & Cohen, 1999; Putnam & Borko, 2000). Borko (2004) has categorized professional development innovations into four phases that focus on individual teacher knowledge and practices, on group dynamics of professional communities, on individual and group records of practice, and on the simultaneous coordination of individual and group learning. She calls for more studies that explore the simultaneous learning of individuals and the group.

How can the professional learning of individuals and the group be investigated? How can relationships within professional development settings be described? To provide a theoretical framework for my study, I turn to the historical writings of Martin Buber and David Hawkins.

Relations of *I-Thou* and *I-It*

The philosophical ideas of Buber (1923/1970)[4] on relations inform my thinking about interactions in professional development settings. He proposes that humans enter into relationships with their whole beings. In describing the nature of these relationships, he offers what he calls two 'primary words' or states: *I-Thou* and *I-It*. These arise in the three spheres of our lives: with nature, with humans, and with spiritual beings. Although the main thesis of his writing is that meetings with one another are reflections of relationships with God, I focus here on his discussion of the nature of *human* relationships.

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[4] There are two main translations of *I and Thou*: Smith's translation in 1958 and Kaufman's translation in 1970. In this dissertation, unless otherwise specified, references are to the 1970 version. All translations in this chapter are from Buber (1923). In Kaufman's translation, he replaces 'Thou' with 'You' in an effort to remove the formal and biblically remote form of address that often conjures up an immediate connection to God. He chooses 'You' to represent direct, spontaneous, and unpretentious human relationships. Nevertheless, in my writing, I use 'Thou' to point to the significance of our relationships with others and to encourage a pause while we contemplate this sacredness. In both cases, Buber's intent to accentuate the exclusivity of this relationship is communicated.

What are these relations of *I-Thou* and *I-It*?[5] According to Buber, there are two basic descriptions of our world, expressed in these word-pair combinations. The world of experience belongs to the primary word *I-It*. The primary word *I-Thou* determines the world of relation. Only *I-Thou* relations are enacted with our whole beings. Here is a further distinction. For Buber, those who experience are not entering into relationships, since *It* does not participate. The world of *It* allows itself to be experienced but contributes nothing and nothing happens to it. In contrast, relations are actual: the relation acts on me as I act on it. Actual relations are dialogic, reciprocal, mutual meetings. Actual relations are genuine and authentic, developing through enactive encounters.

However, I could interpret Buber's terms of *I-Thou* and *I-It* as subjective or personal relations and objective or detached relations respectively. Certainly, I connect his notions of *I-Thou* with possible relations in professional development settings and will do so explicitly shortly. However, after explaining the differences between relations of *I-Thou* and *I-It*, Buber proceeds to show the interrelationships among them:[6]

> This, however, is the sublime melancholy of our lot that every You must become an It in our world. However exclusively present it may have been in the direct relationship—as soon as the relationship has run its course or is permeated by *means*, the You becomes an object among objects, possibly the noblest one and yet one of them, assigned its measure and boundary. The actualization of the work involves a loss of actuality. Genuine contemplation never lasts long; the natural being that only now revealed itself to me in the mystery of reciprocity has again become describable, analyzable, classifiable—the point at which manifold systems of laws intersect. And even love cannot persist in direct relation; it endures, but only in the alternation of actuality and latency. The human being who but now was unique and devoid of qualities, not at hand but only present, not experienceable, only touchable, has again become a He or She, an aggregate of qualities, a quantum with a shape. Now I can again abstract from him the color of his hair, of his speech, of his graciousness; but as long as I can do that he is my You no longer and not yet again.
>
> Every You in the world is doomed by its nature to become a thing or at least to enter into thinghood again and again. In the language of objects: every thing in the world can—either before or after it becomes a thing—appear to some I as its You. But the language of objects catches only one corner of actual life.
>
> The It is the chrysalis, the You the butterfly. Only it is not always as if these states took turns so neatly; often it is an intricately entangled series of events that is tortuously dual. (pp. 68-69, italics in original)

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[5] Buber (1923/1958) states that *I-Thou* and *I-It* "do not signify things, but they intimate relations" (p. 19). However, throughout the rest of his writing he does not refer to *I-It* as a relation. In contrast to Buber, I have chosen to describe both *I-Thou* and *I-It* as relations. This reflects my notion that objects and ideas, such as mathematics, can be encountered with our whole beings. Also problematic is the term *Thou* because it can point to the singular Thou or the plural Thou. Therefore, when I refer to relations of *I-Thou*, I mean multiple relations among many Thou. The relation of *I-Thou* indicates the singular Thou.

[6] I have grappled with my commitment to use gender inclusive language. After crafting several alternatives, I was unsatisfied with the passage. I recognize that this manuscript was written in a different historical era and rather than interrupt the rhythm of the text, I have decided to quote directly from the original translation. The philosophical cadence and poetic expression are qualities I value and wish to retain.

My interpretation of his writing is that the authentic meeting between *I* and *Thou* is intense and impossible to sustain. One thread of this study is to explore the possibility of sustaining this in an inservice setting. The movement throughout states of *I-Thou* and *I-It* are less clear to me.

Where does our relationship with the subject of mathematics fit within this perspective? If I name mathematics as *It*, how can *It* become *Thou*? Encountering mathematics, a discipline steeped in traditions of abstract objectivity (Davis, 1984; Ernest, 1991; Sinclair, 1971), as *Thou* seems to offer a challenge to a traditional perspective of mathematics as *It*, as always *It*, and as nothing but *It*. Again, how does *It* become *Thou*? How does mathematics become known, encountered, and related to *Thou*?

An element of the explanation for me comes from Buber:

> As he beholds what confronts him, its being is disclosed to the knower. What he beheld as present he will have to comprehend as an object, compare with objects, assign a place in an order to objects, and describe and analyze objectively; only as an It can it be absorbed into the store of knowledge. But in the act of beholding it was no thing among things, no event among events; it was present exclusively. It is not in the law that is afterward derived from the appearance but in the appearance itself that the being communicates itself. That we think the universal is merely an unreeling of the skeinlike event that was beheld in the particular, in a confrontation. And now it is locked into the It-form of conceptual knowledge. Whoever unlocks it and beholds it again as present, fulfills the meaning of that act of knowledge as something that is actual and active between men. But knowledge can also be pursued by stating: "so that is how matters stand; that is the name of the thing; that is how it is constituted; that is where it belongs." What has become an It is then taken as an It, experienced and used as an It, employed along with other things for the project of finding one's way in the world, and eventually for the project of "conquering" the world. (pp. 90-91)

For Buber, the acts of beholding and of being known exclusively describe the shifting state from *I-It* to *I-Thou*. This is how a being becomes a *Thou*. Can mathematics be considered a being? Do Buber's ideas offer the possibility of mathematics becoming unlocked from "the It-form of conceptual knowledge"? Or does this shift in relations occur solely between humans? This seems to be the focus of his claim.

If relations are mutually interactive, how does this change of states from *I-It* to *I-Thou* act on *I*? Buber distinguishes the *I* of the primary word *I-Thou* from that of the *I* in *I-It*:

> The I of the basic word I-It appears as an ego and becomes conscious of itself as a subject (of experience and use).
>
> The I of the basic word I-You appears as a person and becomes conscious of itself as subjectivity (without any dependent genetive).
>
> Egos appear by setting themselves apart from other egos.
>
> Persons appear by entering into relation to other persons.
>
> One is the spiritual form of natural differentiations, the other that of natural association.
>
> The purpose of setting oneself apart is to experience and use, and the purpose of that is "living"—which means dying one human life long.
>
> The purpose of relation is the relation itself—touching the You. (pp. 111-113)

Returning to Buber's claim that *I-Thou* involves my whole being, and that *I-It* never does, it would seem that the *I* of the word *I-It* is fragmented. This implies, as Buber asserts, that wholeness only occurs within the context of human relations. However, when I reconsider my memory of *Mutual Learning of Mathematics* (see p. 15) presented at the beginning of this chapter, I wonder how mathematics as beautiful and engaging is encountered? I am confronted by the patterns and relationships, and the mathematics seems to reveal itself. *I* act on *It* and *It* acts on me. I think I am in relation to the mathematics; I think this mathematics has become a *Thou*. Can mathematics become personable? I claim that *I-It* can also be a relation. Buber does not allow for this possibility and thus seems, to me, to miss an important dimension of relating.

Below, I offer a diagram of how his description of relations might frame professional development in mathematics education. In this representation, I have replaced *I* with Teacher Educator[7] and *Thou* with Teacher. The separation between these relations is indicated by a broken line, while the possibility for movement between states is shown by arrows. One arrow is dotted to demonstrate that movement from *It* to *Thou* is not always possible.

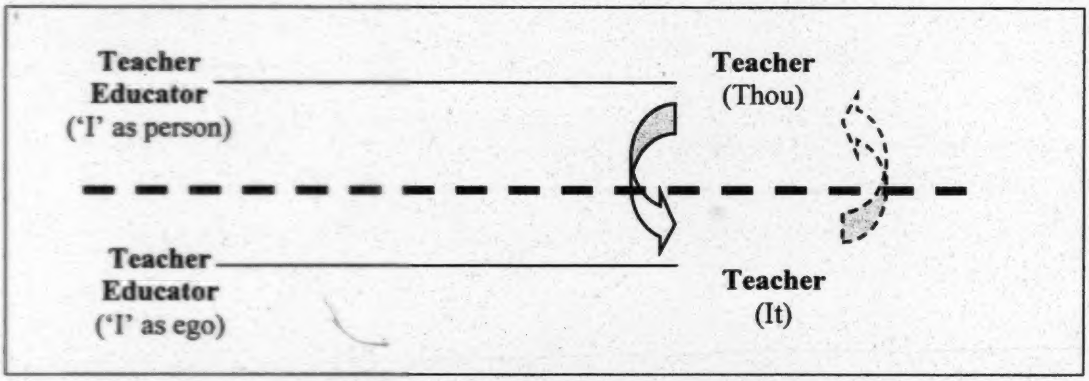
Teacher Educator ('I' as person)
Teacher (Thou)
Teacher Educator ('I' as ego)
Teacher (It)

Figure 2-1. Relations of *I-Thou* and *I-It* in professional development settings

As shown in the diagram, the relationship between the teacher educator and the teacher can be described in two ways. When there is authentic meeting, the teacher educator and teacher enter into a dialogic and mutual relationship with their whole person. Each acts on the other. This relation allows for professional learning to be viewed as an adaptive process where knowledge is co-produced. Consistent with my experience of synergy and with emerging theories of learning, professional relations of *I-Thou* are believed to be on-going interactions between individuals in social, cultural, collaborative, and participatory contexts (Loucks-Horsley, Hewson, Love, & Stiles, 1998). Such relations are the focus of my research (see p. 12).

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[7] Within the context of professional learning, several terms are used to explicate the visible relationships. Throughout this dissertation, I refer to teacher educators as those who teach teachers, either in an inservice or preservice context. I have deliberately paired the two words to point to the relationship as a teacher of teachers. As I interact with teachers in a professional development context for my study, I view myself as a teacher educator. In addition, I am also a researcher, as I study and interpret the interactions of the participants.

The other relationship between the teacher educator and the teacher is described by *I-It*. Here, the teacher educator is separated from the teacher and is experienced or used. These professional relations depend on the teacher educator to train, build expertise, or improve teachers' competence, skill, and knowledge. Teacher educators relate to teachers by providing answers in the form of information to a 'consumer' of such knowledge. In my experiences of *Disconnection* (see p. 1) and *Ambivalence* (see p. 4), these notions of professional relationships were apparent. Experts presented information and distributed materials and learning was viewed as acquiring knowledge.

As illustrated, it is inevitable that teachers will be experienced as *It* because according to Buber, relations of *I-Thou* cannot be sustained indefinitely. Certainly, my experiences described in the Prologue reflect this phenomenon, as I was viewed by various teacher educators as an *It* and as I so viewed teachers. This has strong implications for professional learning as I consider how a relation of *I-Thou* is sustained.

While Buber related *I-Thou* to *I-It*, it is the *I-Thou* relationship that is celebrated. He believed that we can only grow and develop as we live in relation to others:

> Whoever stands in relation, participates in an actuality; that is, in a being that is neither merely a part of him nor merely outside him. All actuality is an activity in which I participate without being able to appropriate it. Where there is no participation, there is no actuality. Where there is self-appropriation, there is no actuality. The more directly the You is touched, the more perfect is the participation. (p. 113)

Thus, the quality of the *I-Thou* relation can be considered paramount to learning.

I am still puzzled by how the teaching and learning of mathematics can be interpreted within this perspective. Assuming that mathematics can become known in a relation of *I-Thou*, I could make similar substitutions of *I* by Teacher Educator and *Thou* by Mathematics, or of *I* by Teacher and *Thou* by Mathematics. Clearly, this diagram fails to describe the relations between the teachers and mathematics and the multiple relations among teachers in the group. If I switch the labels of Teacher Educator and Teacher, thus marking Teacher as *I*, my relation to the mathematics as teacher educator is obscured. Departing from Buber's primary focus on the *I-Thou* relationship, David Hawkins' work explicates the importance of relating to *It*. Expanding on Buber's ideas, this provides a way for me to look at relations among *I*, *Thou*, and *It* simultaneously.

The Triad *I-Thou-It*

When I first read Hawkins' (1974) writings, I was fascinated by his investigation of parent-child and teacher-child relationships. I discuss reasons for my interest in Interlude 3. I focus here on his exploration of the relationships among children, teachers, and mathematics. In an essay entitled *I, Thou, and It*, Hawkins refers to several ideas that lead me to believe that he is familiar with Buber's writing, although he does not explicitly acknowledge this source. The title of his essay points to Buber's notions of relations and he reiterates Buber's claim by stating, "Without a Thou, there is no I evolving" (p. 52). Also, he refers directly to the *I-Thou* relation. Calling implicitly on Buber's notions, he describes the following triad of relationships necessary for children's learning.

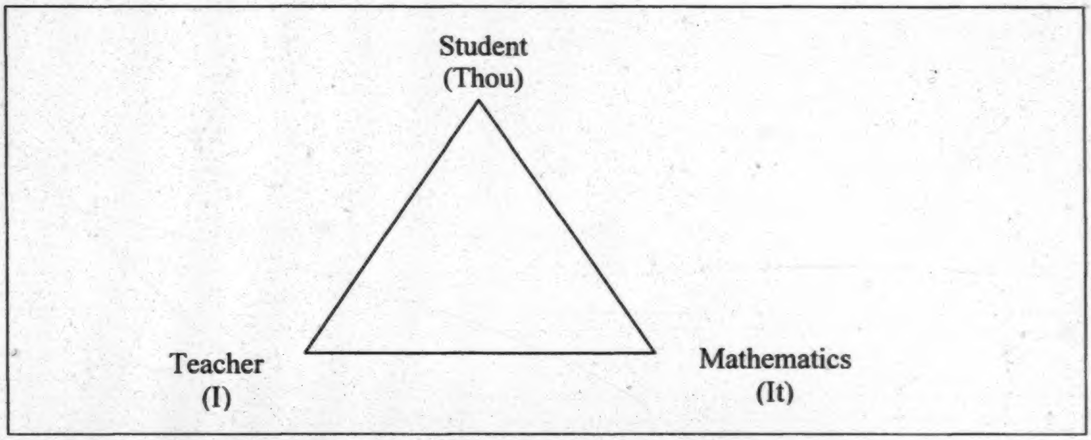

Figure 2-2. Relations among *I*, *Thou*, and *It* in classroom settings

What is noticeably different from Buber's proposal is Hawkins' emphasis on the relationships with the mathematics. Three vertices represent *I* (the teacher), *Thou* (the student), and *It* (the mathematics). He suggests that the third vertex of the triangle acts on the other two, entering into "the pattern of mutual interest and exchange" (p. 50). Here, the interaction between these three "beings" is described in Buber's terms of *I-Thou* relations of mutual encounter. Hawkins states that you cannot separate *Thou* from the non-human or the human world. He explains these relationships in this way: "Without a Thou, there is no I evolving. Without an It there is no content for the context, no figure and no heat, but only an affair of mirrors confronting each other" (p. 52). Without an *It*, *I-Thou* is merely a reflection of relations.

Three dyads, two of which are part of Buber's notion of relations, are apparent within this triad: *I-Thou*, *I-It*, and *Thou-It*. Within a classroom setting, these dyads are represented by the edges of the triangle. Here the relationship between the teacher and student is always an *I-Thou* relation and the relationship between the teacher and the mathematics is different from the relationship between the student and the mathematics.

What seems problematic is Hawkins' apparent assumption that there is one mathematics. However, it is possible to have very different mathematics. Describing a classroom-based inquiry into properties of soap bubbles, Hawkins emphasizes the importance of the teacher in transforming the spontaneous interest of the child. He refers to this process as providing an external feedback loop. But it is the soap bubbles that encourage an encounter with mathematics. He writes:

> The teacher has made possible this relation between the child and "It", even if this is just by having "It" in the room; and for the child even this brings the teacher as a person, a "Thou," into the picture. For the child this is not merely something which is fun to play with, which is exciting and colorful and has associations with many other sorts of things in his experience: it's also a basis for communication with the teacher on a new level, and with a new dignity. […] Then the child comes alive for the teacher as well as the teacher for the child. They have a common theme for discussion, they are involved together in the world. (p. 57)

The mathematics for the student might be geometries and patterns of shape and space. The mathematics for the teacher might include pedagogical dimensions of how the student is making sense of the mathematics and how the student can be prompted to make deeper connections.

According to Hawkins, the richer the encounter with *It*, the stronger the relationship between the teacher and child. In fact, Hawkins goes further by claiming, "Some children are *only* able to develop humanly by first coming to grips in an exploratory and involved way with the inanimate world" (p. 61). It seems if I ignore the importance of *It* in the study of relations, this will limit an analysis of the growth and development of individuals.

If I view professional development contexts as classrooms, Hawkins' ideas translate easily. His description of the teacher fits with teacher educator, the student is reframed as the teacher, and the *It* becomes the professional development task. Buber's mutual relations of *I-Thou* are expanded in this perspective to include *I-It* relations. Three dyads of *I-Thou-It* act on each other and are necessarily interdependent.

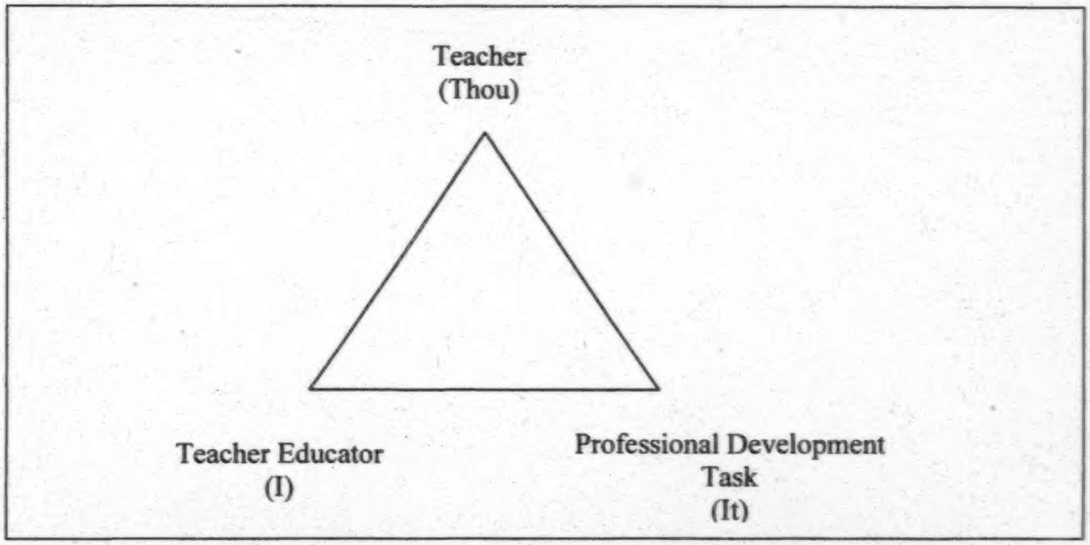

Figure 2-3. Relations among *I, Thou,* and *It* in professional development settings

I find the distinction that Buber makes between *experiences*[8] and *encounters* helpful when describing relations within professional development settings. According to him, *I-It* relations are one-way experiences where the *It* does not participate in any way. *I-Thou* relations are encounters, meetings between people. Here the relation is dialogic and participatory Again, I assert that these encounters can be between people and content, in this case mathematics. Expanding his ideas, I can express the quality of relationships as experience or encounter or on a continuum between these two poles. In the Prologue, I deliberately used the terms *experience* to refer to those situations where the *I-Thou* relationship appeared weak or non-existent and *encounter* where *I-Thou* relations seemed strong. When Hawkins claims that development depends on the quality of the *I-It* relation, I could interpret this as describing encounters with mathematics. Thus, when I view *It* as an object, I experience and use it; when *I-It* shifts to *I-Thou*, I respond with my whole being and participate in an encounter. It is this quality of the *I-Thou* relation that determines the individual growth and development. I claim this relation can occur between elementary teachers and mathematics.

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[8] It is easy to confuse this use of the term *experience* with that which is commonly used to denote such things as teaching experience. Clearly, the English term *experience* is problematic in this context as it can point to *I-Thou* relations within school settings. While I have experimented with the use of the German noun E*rfabrung* and the verb E*rfabren* instead of *experience* to signify Buber's notion of the *I-It* relation, this becomes cumbersome. I leave it to the reader to discern my intention when interpreting the term *experience* within this text.

My interpretation of Buber is that the dyads of *I-It* and *Thou-It* form the world of experience and *I-Thou* constitutes the world of relations. However, Hawkins claims that relationships with *It* are mutual and provide the context for learning. Relations are more than an affair of mirrors. I suggest that all three dyads can be encountered as the mutual relations Buber describes as *I-Thou*.

The image in Figure 2-3 is helpful when describing relationships among individuals. Many of the memories of my professional development experiences presented in the Prologue, as well as my encounter with mathematics recalled at the beginning of this chapter, could be described fruitfully in terms of *I-Thou-It*.

One limitation of this representation of relations in professional development settings is the ambiguity of the *I*. In the diagram, the *I* is shown as the teacher educator. However, in my study, there is also the *I* of a researcher. Huberman (1999) investigates the impact of what I call sustained *I-Thou* relations on researchers. He suggests that this interactivity benefits both teachers and researchers, by allowing researchers to refine conceptual frameworks and communicate their findings in a way that is helpful to the professional practice of teachers. For me, it is important that I relate to teachers, that I do not become disconnected from the classroom, and that I am not perceived as living solely in the "Ivory Tower." I want my research to relate to the practicalities of teaching and do not want to be perceived as an isolated and remote intellectual who remains aloof and demonstrates an attitude of academic elitism.

An *I* as a mathematician is also embedded in the professional development setting. For teachers who do not relate to mathematics, this *I* can be perceived in negative terms. Relations between *I* and *Thou* can become distant if the closeness to the mathematics is perceived as an impediment to the teaching of mathematics. In other words, some teachers may feel that my success in mathematics limits my ability to understand how others might struggle with it. Being viewed as a mathematician can accentuate differences between *I* and *Thou*.

The *I-Thou* relation is important to me because I want to "reframe the goal of research from one of primarily informing the practitioner to one of jointly constructing knowledge through shared activity" (Huberman, 1999, p. 291). For me, there is a need to be viewed like a teacher and yet I am no longer an elementary teacher. There is ambiguity between my resemblance to teachers and my dissimilarity from teachers. How do I direct and lead as a teacher educator without being seen as an authoritative expert in both mathematics and teacher education? How can I prompt robust *I-Thou-It* relations?

Possible Relations in Professional Development Settings

A traditional training perspective of professional development emphasizes the consumption of knowledge. This view was evident in most of my memories described in the Prologue and can be described using Buber's term of experience. In contrast, inquiry approaches focus on the critique and production of knowledge. Many teacher educators recognize the importance of collaboration and learning within a community and seek to change the relationship a teacher has with the mathematics. In many cases, these can be described as encounters, since relationships among teachers, the teacher educator, and the mathematics seem robust.

Within a classroom context, the *It* is the mathematics. However, there is much less agreement of what the *It* is in professional development settings. Different researchers offer different *Its*. In order to consider alternative possibilities, I present several examples of systemized inquiry approaches to professional development in elementary mathematics.[9] Many of these innovations attempt to offer possibilities for encounters rather than experiences. In other words, opportunities for a strong triad of *I-Thou-It* are apparent in these instances. Relations among *I*, *Thou*, and *It* form the framework for my description and analysis of these programs.

Cognitively Guided Instruction (CGI)

Based on cognitive research, Carpenter and Fennema (1992) suggest that primary children's understanding of mathematics prior to entering school is informal and intuitive. They believe that these intuitive approaches to solving problems through the basic operations of addition, subtraction, multiplication, and division can inform teachers' pedagogy. From observations of children's mathematical activity, Carpenter, Fennema, Franke, Levi, and Empson (1999) classify these intuitive problem-solving processes and identify problem types and children's solution strategies. This classification structure forms the content, the *It*, of professional development tasks.

Assuming that primary teachers need to understand students' mathematical thinking, Carpenter and Fennema designed a four-year project where explicit frameworks for interpreting students' strategies for solving problems were presented. Participants were primary teachers from four different schools. During the first and second years, teachers participated in a series of workshops offered by facilitators trained by the researchers. Teachers were introduced to a research-based model of how primary children understand basic number concepts and operations and were presented with the classification structure. By viewing student work and videotapes of students engaged in problem solving tasks, teachers analyzed students' mathematical thinking. Teachers participating in the project learned about their students' prior knowledge of mathematics and investigated research findings of how children learn specific mathematical concepts.

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[9] While these programs represent only a sample of the innovative approaches to professional development in elementary mathematics evident in the research literature, they are the most prevalent and represent long-term research initiatives. Other examples of professional development initiatives include the Purdue Problem-Centered Mathematics Project (Cobb et al., 1991; Wood & Sellers, 1997), the Developing Mathematical Ideas seminar (Cohen, 2004; Schifter, Bastable, et al., 1999), and the Integrating Mathematics Assessment project (Gearhart et al., 1999; Saxe, Gearhart, & Seltzer, 1999; Kazemi & Stipek, 2001).

The researchers did not provide teachers with instructional materials. Instead, teachers were encouraged examine their practice, create student tasks based on their evolving understanding of students' thinking, and observe their own students to validate the research model. A project staff member and a mentor teacher were assigned to each of the four schools and provided support to teachers as they translated their new knowledge into their practice. The staff member and mentor teacher attended all workshops and visited each teacher's classroom once a week. They observed the implementation of the curriculum and provided feedback to encourage reflection and ongoing inquiry. Reflections were shared with the group during 14 workshops throughout the second year. During the third and fourth years, the number of classroom visits and workshops decreased substantially. Teachers were not presented with any new information but continued to observe and analyze their students' work and behaviour.

Researchers implementing CGI programs propose that "an understanding of students' thinking can provide coherence to teachers' pedagogical content knowledge and their knowledge of subject matter, curriculum, and pedagogy" (Carpenter, Fennema, & Franke, 1996, p. 3). In the CGI approach to professional development, the *It* is student learning of mathematics. Teacher learning is focused on analyzing student artefacts of mathematics. Relations between teachers and the mathematics appear to be strong as the program presents a research-based model of student learning. Teachers are prompted to create instructional materials that reflected the research and this process expands possibilities for relationships with the mathematics.

Teachers portrayed their initial relationship to the CGI task as challenging, as evident in Terese Kolan's description:

> I first learned about CGI many moons ago. But it took a while. It wasn't an easy task because you're learning about all the different types of strategies that children can use. You're also learning about all the different types of problems there are. And so you're trying to put that all together. And then you're trying to make it usable for you. And I remember feeling very overwhelmed, going, "Okay, I've got to look at this problem, now I've got to look at this type of strategy." And it wasn't an easy task. I had to go over it a lot. And then after a while it just becomes a part of you like everything else does. After you use it and you have it, it becomes easier. And it feels right. It's more comfortable. (Carpenter, Fennema, Franke, Levi, et al., 1999, p. 85)

Other teachers described their unwillingness to abandon textbook-centred instruction. As Chambers and Hankes (1994) report, teachers were more confident when relying on a textbook, they were uncertain about knowing if the curriculum was being taught, and they viewed problem solving as less of a priority than completing pages in the textbook. However, as comfort levels rose and confidence increased, teachers reported less dependence on the textbook. One teacher explained that "I am so comfortable without a basil text now. I envisioned that being hard at the beginning, but the text just ties you down. It stifles you" (p. 291). Relations to the *It* of CGI needed to develop over time.

Relations of *I-Thou* were not explicitly investigated in the research. However, references to interactions indicated strong *I-Thou* relations. Ann Badeau described her experience of teacher autonomy:

> When you said, "You're sort of on your own, we will help you out, but you have free reign," it was clear that we were to make our own decisions, and I found that scary. But then after a while, I thought, they are really treating us like professionals, like we know something, instead of spoon-feeding us. (Carpenter, Fennema, Franke, Levi, et al., 1999, p. 95)

Because instructional materials were not specified, teachers were expected to interpret the model within their own classroom setting. Mentors were able to interact with the teachers as they made mutual sense of their observations. The design of the project inherently encouraged strong relations of *I-Thou*. I suspect teachers who were not encountering positive *I-Thou* relations would opt out of the project. The description of classroom observers as mentors implied that support and guidance would be offered by a trusted person.

Relations of *I-Thou-It* are described in Huberman's (1999) investigation of the interactivity between teachers and researchers:

> If something is to happen in the sense of a shared endeavour, the strongest reciprocal effects are likely to come from a clearly defined project, one that spins out over time and where there is something on the line for both parties. This was one of the findings of the CGI program. Here, the evolutionary, cumulative nature of the project helped to reduce the social distance among practitioners, researchers, graduate students, and teachers in the project. Their roles remained different, but their focus was common and concrete: the notion that teaching is not independent of, but rather contingent on, what particular pupils know and are able to do. This focus, in turn, was driven by cognitive and developmental issues rather than by the logical-theoretical structure of mathematics and its acquisition. Such a focus relies on close observation, followed by inferences drawn by multiple investigators. (p. 300)

It is apparent that the CGI program offered an opportunity to prompt teacher reflection on students' mathematical thinking. Research indicates that teachers modified their beliefs about teaching and learning mathematics by linking what they were learning in the workshops to what they were observing in their classrooms (Chambers & Hankes, 1994). However, time remained a limitation. Teacher change was observed over a period of four years. During the first two years of the project, many teachers resisted this *It*.

For my own research, I incorporated several features of CGI. By having a clearly defined project that allowed for flexibility in its implementation, I offered a "sense of shared endeavour." The triad of *I-Thou-It*, apparent in the design of the CGI project, was also considered in my own study. Envisioning myself as a mentor, facilitator, and researcher in terms of the CGI project was helpful as I sought a participatory encounter with the teachers. Although I did not focus on students' thinking, I included classroom observations and feedback in the implementation phase of my project. Like the teachers in the CGI program who were encouraged to focus on the prior experiences and intuitive understandings of their students, I strove to attend to teachers' experiences and prior knowledge of mathematics as a basis for our tasks.

There were several features of the design of CGI that I chose not to include. I had several limitations with my study, one of which was time. I did not have time and resources to engage in a four-year project. Also, instead of emphasizing teaching through story problems, I chose to focus on teachers' understanding of the nature of mathematics through historical story. While I did not choose to emulate the content of CGI, I integrated some of the design features into my study to prompt robust *I-Thou-It* relations.

SummerMath for Teachers

Based on constructivist approaches to learning, Schifter (1993) suggests that teachers need to experience mathematical processes as they actively construct their own mathematical knowledge. By reflecting on these processes, she hopes that teachers will learn to think mathematically. Her approach to professional development seeks to address teachers' insufficient understanding of mathematics and the limitations this places on the promotion of mathematical exploration and debate in their classrooms.

SummerMath for Teachers, an intensive summer institute, is based at Mount Holyoke College. This inservice program introduces teachers to constructivist theory as teachers work on mathematics in order to improve their content knowledge and understanding of mathematical processes (Schifter, 1993; Simon & Schifter, 1991). Because the content of *It* is not well specified, there is some flexibility in how the approach is implemented.

One such example was initiated by teachers' requests for a course taught in a way teachers were being taught to teach. This program was developed and offered to 49 teachers as a weekly three-hour class. The first thirty minutes of the class was devoted to discussion on questions arising from previous classes and readings. The remainder of the time was committed to small group investigation of a mathematics task and whole class sharing of discoveries and questions. These tasks focused on whole number operations, fractions, decimals, measurement, and geometry and many emphasized the meaning of symbols and properties of the number system. Assigned homework included problems for further exploration, readings, and journal reflections.

By reflecting on their own experiences of learning, teachers were given the opportunity to explore what learning mathematics might mean for their students. The emphasis was on personal constructions of knowledge and teachers were required to write about what they were learning and submit journals once a month. Thus, teachers were encouraged to become reflective practitioners (Schön, 1983).

The course focused on the development of mathematical reasoning powers and relations to *It* seemed strong. However, unlike the CGI project, the mathematics was not systematically prescribed as part of the project. This may have contributed to less robust understandings of mathematical operations. One teacher commented:

> If I keep playing, the inter-relationships between the operations will become more and more tangible for me. I can say—subtraction is the reverse of addition—but that's a limited way of looking at that operation. Besides, I guess I don't even know what "is the reverse of" means—AND what the implications may be in various situations. (Schifter, 1993, p. 274)

Since the inception of SummerMath for Teachers, other projects addressing specific areas of mathematics content evolved. Conducted by SummerMath for Teachers, one such example was the Teaching to Big Ideas (TBI) project (Schifter, 1998). This project involved 6 teacher educators and 36 teachers who participated in the two-week summer institutes. In addition, the project design included biweekly after-school seminars and biweekly classroom visits by a staff member. Lessons created by the project staff members focused on the interconnections and relationships among specific domains such as number systems and whole number operations, fractions and decimals. Another project that evolved from the SummerMath for Teachers project was the Developing Mathematical Ideas (DMI) seminar. Five modules emphasizing themes of number concepts, number operations, geometry, and data management were designed for eight three-hour sessions. In each of these projects, the relation to *It* appeared strong.

One strength of the SummerMath for Teachers program seemed to be the opportunity for reflection on their own mathematical processes, which allowed some teachers to deepen their understanding of the nature of mathematics. Many teachers commented on their growing acceptance of processes of frustration and puzzlement and their ability to set aside the sole goal of getting the correct answer. One teacher wrote:

> I have discovered how difficult it can be to explore an idea that has been a misconception. The basis on which it is founded can be so sophisticated that it's not easily undone. Because it has already managed to survive a test of time, it's hard to undo that entire structure or network that has been built around the misconception. My own ideas of ratio have been challenged several times, and are still not entirely clear to me. Ratios can function in so many roles that it's hard for me to even identify the extent of my misconception. It can be extremely frustrating, which has given me a far better understanding of the feelings evoked [in] students, when struggling with misconceptions. The process is as much emotional as it is intellectual, as it calls your own competence into question. There is a period of chaos when a concept has been challenged, knowing that it doesn't fit but not knowing how to make it right. (Schifter, 1993, pp. 277-278)

Many of these teachers worked through their frustration and experienced an increased confidence in their own abilities as mathematical thinkers. Their relationship with mathematics changed. Another teacher commented:

> Because of the confidence and new perspectives towards problem solving this course has given me, I was able to follow (albeit gingerly) a line of thought that I never would have attempted to attend to before—and my reward was a personal immediate experience in which I was conscious of the living power of mathematics thought—not someone else's account, but mine! (Schifter, 1993, p. 280)

Similar to relations of *I-Thou* in CGI settings, SummerMath programs appeared to encourage strong relations between the teacher educator and the teachers. This seemed to arise out of the approach to teaching, as explicated by one teacher:

> I think that being so highly respected as a learner has…been important to me. My mathematical knowledge does not extend beyond high school level, but [in this class] I never felt that was an impediment. Classes usually have a hierarchy of respect, with the most knowledgeable at the top. That was noticeably absent in this class, where your own thinking was far more important than assimilated level of knowledge. (Schifter, 1993, p. 281)

In the DMI project, special attention was given to the role of the facilitator as documented in the facilitator's guide. Assuming that "where the nature of the activity being aimed for sharply departs form current practice, most groups will not find their way without determined and knowledgeable leadership" (Schifter & Lester, 2002), the role of the facilitator was to pose questions and shape the discussion. Three directives were given: facilitators must understand seminar content; facilitators must focus on the program's learning goals for the teachers; and facilitators must attempt to understand teachers' perspectives (p. 8). The facilitators were strongly active in the interactions between *I* and *Thou*. In an effort to provoke deeper reflection, there was a risk of jeopardizing the *I-Thou* relation by communicating a disregard for teachers' ideas. Facilitators were viewed as being responsible for the learning of the teachers but were reminded that this form of facilitation demanded "a stance of respect for and commitment to the participants in the seminar and the ideas to be explored" (p. 24).

Research findings suggest that the SummerMath for Teachers project offers opportunities for teachers to grapple with the nature of mathematics as they improve their own content knowledge. Schifter (1998) found that teachers participating in her project

> developed new conceptions of the nature of mathematics and a heightened sense of their own mathematical powers. As learners of mathematics, they experienced a new kind of classroom, one in which the social character of doing mathematics was realized, and in which student thinking took center stage. In addition, these mathematics lessons became occasions for teachers to reflect on their own process of learning and to consider those features of the classroom that supported or hindered that process. (p. 83)

My research was informed by some of the features of this project. I wanted to provide opportunities for teachers to deepen their understanding of mathematics. When we were deciding on a topic for investigation, I suggested that we study number systems, a content area of the elementary school curriculum. I believed that teachers needed the opportunity to engage in reflective writing. Therefore, the teachers in my study were encouraged to write responses to our readings and discussion in reflective journals that were then shared with one another. As a teacher educator, I tried to pose questions that would provoke deeper understandings about the nature of mathematics.

I chose not to take responsibility for the teachers' learning but, instead, I responded to the emerging issues taken up by the teachers. It was very important to me that I respect teachers' ideas and I think this was communicated as I sometimes set aside my agenda for the meetings. I viewed my facilitation as a negotiation among all of us as equal participants.

Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR)

Stein, Smith, and Silver (1999) present a new paradigm for professional development. Features of this new approach include a focus related to the work of teaching and grounded in the content of teaching and learning. The development of communities of professional practice and collaboration with experts outside the teaching community are emphasized.

This paradigm informed the design of the QUASAR project. Housed at the University of Pittsburgh, researchers studied nearly 500 classroom lessons of mathematics. Working in six urban middle schools over a period of five years, they selected prevalent cases and analyzed the tasks being offered to students. The tasks were organized into four levels of cognitive demand: memorization tasks, procedures without connections tasks, procedures with connections tasks, and doing mathematics tasks. These categories formed a Mathematical Tasks Framework. Teachers were then given opportunities to use this framework to analyze the tasks they were offering to students, to reflect on their instructional practice, and to share their ideas with colleagues.

Instructional cases were created from this process. These cases were offered to teachers and teacher educators as a study tool to prompt discussion. This program for professional development, described by Stein, Smith, Henningsen, & Silver (2000), focused on the practice of mathematics instruction through the study of cases. Researchers presented common cases of mathematics instruction where images of reform teaching, discussion questions, and a framework for task analysis formed the agenda for teacher learning. Each case study focused on mathematical tasks and investigated levels of cognitive demand. Teachers were encouraged to design mathematical tasks for their students, implement these in their classrooms, and evaluate their instructional decisions according to the Mathematical Tasks Framework provided by the researchers. Incorporated into the program were opportunities for teachers to observe each other's practice in order to reflect on student learning and cognitive demand. Two dimensions of teacher learning were emphasized: "Exposure to our cases aims to assist teachers to develop (1) an understanding of mathematical tasks and how their cognitive demands evolve during a lesson and (2) the skill of critical reflection on their own practice guided by reference to a framework based on these ideas" (p. 33).

The *It* in this context was mathematical pedagogy. Relations to the mathematics seemed strong as teachers were focused on the creation and analysis of mathematical tasks for student learning. Some teachers found that by designing mathematical tasks, they learned much about mathematics (Smith, 2000). Although other teachers may not have increased their own content knowledge in the process, they had ample opportunities to reflect on their instructional practices. Teachers made some pedagogical changes but seemed "underprepared and too overwhelmed for the task of developing curriculum" (Stein, Smith, & Silver, 1999, p. 258).

In the initial years of the project, teachers experienced strong relations of *I-Thou* as they participated in the professional development tasks. There was "tremendous cohesion among member of the teacher community" (Stein, Smith, & Silver, 1999, p. 253). However, the relational distance between teacher educators and teachers was evident in some instances. The teacher educators were experts who facilitated and orchestrated the learning of the teachers. Although learning occurred through interactions among teachers, teacher educators, and mathematics, in some cases the quality of these interactions was compromised by the imposition of the expert agenda to influence teacher practice. One teacher commented:

> We needed the staff development that we had at the beginning. It was absolutely critical. And I think had that not happened, we wouldn't have been as successful. But the thing I'm pointing to is to where our needs began to change a little bit. We were still back doing more of that other stuff, and we just didn't see that as big enough. I think one point for staff developers to realize in the future is that there comes a point, especially in a long-term thing like this, that you let your teachers go to do their thing. You just [need to] be there to support them. You…back off a little bit. (pp. 251-252)

The authors stated that "at no point in the project did decisions regarding professional development reflect the perspectives of both the resource partners and the teachers" (p. 251). This contributed to teachers feeling that they were not respected and their expertise was not valued. During the fourth and fifth years of this project, the teacher educator withdrew because of a perceived inability to co-design a professional development plan and teachers implemented their own agenda.

The researchers concluded that there were many challenges to developing learning communities. They experienced a lack of cohesive focus and suggested that teachers needed to "participate in a common set of activities as the glue that holds a community together" (p. 266). Respecting the knowledge of teachers was important:

> By providing opportunity for all teachers to become 'expert' in the same recognizable way of teaching mathematics and providing the language for talking about the program to one another and to teachers outside their immediate community, it provided a connective tissue that reached across teachers and classrooms. (p. 266).

My research was impacted by this project. To meet the many challenges they faced, the researchers suggested that the focus should be on developing teachers as individuals as well as on growing self-sustaining learning communities. I attempted to attend to both of these dimensions in our professional development setting. Issues of researcher control and teacher autonomy were addressed and I decided to explicitly emphasize the importance of flexibility and communication as we co-planned agendas for each meeting. I was aware that my guidance would be more necessary at the beginning of the project but that I would prompt others in the group to share their expertise as the project progressed. It became very important for me to structure a professional development task that would allow teachers to engage in a common activity but would remain responsive to emerging ideas.

Knowing about challenges faced by other researchers, I was more sensitive to issues arising while learning in a community. Included in my project were opportunities to observe other teachers and to situate our task within a classroom context, notions arising from my study of the QUASAR project. Certainly these were key features of my research design.

Teacher Study Groups

The programs discussed above have strong agendas for teacher change and are designed by teacher educators or researchers. In contrast, a study group is a bottom-up initiative. A study group can be described as "a collaborative group organized and sustained by teachers to help them strengthen their professional development in areas of common interest" (Cramer, Hurst, & Wilson, 1996, p. 7). Relations within teacher study groups, as an inquiry approach to professional development, are dialogical and conversational as participants reflect on their professional practice. The design of a study group prompts the critical examination and experimentation with curriculum reforms. The agenda is collectively agreed upon as teachers collaborate in the project. Sometimes these include discussions of selected readings, engagement in mathematical tasks, analyzing students' work, and observing and discussing a case. Teachers are viewed as meaning-makers (Meyer et al., 1998). Leaders change throughout the study as traditional hierarchy and authority patterns are challenged. The teacher educator participates as a learner not as an expert.

In a study by Crespo (2002), seven elementary teachers participated in a study group that met every three weeks for two and a half hours throughout three years.

Teachers discussed selected readings, engaged in mathematical problem solving that challenged and extended their own understanding of the subject matter, analyzed students' work, and observed each other teach. The project was designed by the teacher educator to strengthen the teaching of mathematics by partner schools and to strengthen the field-based element of a teacher education program. This project was described as a study group by the researcher. However, there was no clear evidence of how the teachers organized themselves or how the agenda was determined.

The *It* in this particular study involved multiple tasks. Teachers were engaged in mathematical tasks and in the analysis of students' mathematical work. This design drew on the innovative approaches of CGI and SummerMath. Within this context, Crespo found that participants encountered multiple opportunities for professional development. They enjoyed learning and felt that their own content knowledge and their teaching practices were positively affected. One teacher explains:

> I've really enjoyed the opportunity to get together and talk/share with other teachers about mathematics. I've seen myself look more critically at how I talk or ask questions to the class during a math lesson. I also have seen myself grow in the area of challenging my students…not being afraid to give them an open-ended story problem. (p. 1442)

Another teacher states:

> I find myself pushing my students to think beyond more. Instead of stopping at solving a problem one way, we talk about alternatives. We also do more thinking/talking about connections to other problems. (p. 1442)

Across different study groups, the *It* will vary in response to teachers' study choices. Relations to mathematics are likely to be strong since there is consensus on the topic of study. This approach to teacher professional development offers flexibility to group members. The *Its* of each of the innovations previously discussed could be incorporated into this approach, as seen by Crespo's integration of research from CGI and SummerMath programs.

I-Thou relations have the potential to become strong since hierarchies and distances between expert and learner are dissolved. No agenda is imposed. Participation is voluntary as this is a teacher-initiated and sustained endeavour. What makes this approach different from professional learning communities (PLCs) is the explicit attempt to respect teacher autonomy. Unlike PLCs, study groups are not top-down initiatives where a leader, usually the principal, guides teachers as they develop statements of commitment in the form of missions, visions, values, and goals focused on student learning (DuFour & Eaker, 1998).

To me, the distinction between study groups and PLCs is significant. However, some researchers propose that they are similar. For example, Tichenor and Heins (2000) described study groups as a strategy for school improvement. Although participation in their study group was voluntary, teachers who were involved earned "staff development points." The researchers did not specify what these were used for but school administrators provided these incentives and rewarded teachers for their participation. Guidelines for success included focusing on what educators considered to be important and discussing study group outcomes at staff meetings. Both teachers and administrators were members of the study group and leadership provided by administrators was encouraged. This particular description of study groups seemed to emphasize school improvement, the main thrust of PLCs.

I believe PLCs and study groups offer relations of *I-Thou-It* in varying degrees. In PLCs, the system is hierarchical and reflects a business approach to education as communities are often described as organizations and students are portrayed as consumers. The emphasis shifts from teaching to improved student learning and schools. However, the teachers are not expected to expand their understanding of mathematics but rather to learn how to teach the prescribed content. Because the focus is on student learning, there is an assumption that learning can be modeled and predicted. In addition, uniformity tends to be stressed as improvement is documented by student performance on standardized tests. This can limit teacher diversity and creativity.

Study groups are intended to promote teacher autonomy. Teachers involved in study groups describe their experiences as collaborative and supportive of change. In one study, Matlin & Short (1991) described this change:

> Changes are evident in every classroom in the building. Students are learning about literature in new and meaningful ways. Teachers are looking at curriculum, learning, and evaluation as powerful collaborative experiences. They're taking responsibility for their own decision making, not simply accepting the words of "experts." (p. 68)

A teacher involved in the study group claimed, "We're changing our basic ways of thinking, not just adding a new activity. This is the scariest and most exciting thinking I have ever done" (p. 68).

I was influenced a great deal by research on teacher study groups when designing my study. What became clear to me was the emphasis on teacher-initiated collaboration. For me, this went further than a desire for learning within a community where all members held a common interest. I wanted to work alongside the teachers, in conjunction with them, co-joined and united in our cooperative endeavour. Thus, I incorporated notions of teacher collaboration into the study's design and tried to encourage a sense of community.

The role of teacher educators within this setting was less clear to me. As a teacher educator, Crespo initiated the study group and was able to negotiate her place among the teachers. However, the inclusion of teacher educators in a study group has the potential to change the intended dynamics of *I-Thou* relations in much the same ways as participants experienced in the QUASAR project of Stein, Smith, and Silver (1999).

Lesson Study Communities

Originating in Japan as a teacher-initiated endeavor and similar to teacher study groups, lesson studies are beginning to emerge in North American conversations about professional development. Takahashi and Yoshida (2004) describe a lesson study as a teacher-led initiative a particular group of teachers collaboratively construct long-term goals for student learning and plan, implement, and observe a lesson designed to further these goals. It is "a form of research that allows teachers to take a central role as investigators of their own classroom practices and to become lifelong, autonomous thinkers and researchers of teaching and learning in the classroom" (p. 438). Stigler and Hiebert (1999) suggest that the study lesson approach to professional development engages teachers in career-long learning as their classrooms become sites for developing teaching-centered theories.

Within this approach, teachers choose a topic and proceed to plan a lesson that focuses on student learning. This lesson is standardized and one teacher volunteers to teach it. While it is being taught, the other group members observe the lesson. The observations are discussed and the lesson is revised. A second teacher volunteers to teach the revised lesson and again teachers observe the lesson and discuss their impressions. This is referred to as a lesson study process (Fernandez & Yoshida, 2004).

Similar to relations of *I-Thou* in study groups, the relations within these communities are strong. All members determine the theme of the study. Students are the focus of the teaching and the examination of practice is thus embedded in the classroom context. Collaborative support among colleagues strengthens these relations as participants learn from one another's teaching experiences.

Relations to *It* are potentially strong. External experts play a role as the group relies on knowledgeable others to provide critical feedback. This expert is invited to the group as an advisor. Although this approach seems to acknowledge the necessity of expert advice, the teachers initiate this relation. There is an implicit focus on research but teacher educators have no participatory role in this community. Although the potential for a learning community to emerge is high, the focus of lesson study is on individual learning.

To me, the nature of the lesson study is cyclical yet it becomes focused as the lesson is revised and its details are fine-tuned. I do not believe that the process expands possibilities; rather it potentially limits creativity as the art of teaching is standardized. Teachers observe the lesson without interacting with the students in the classroom thereby replicating *I-It* relations typical of traditional research approaches. It is possible that diversity within the community might be further diminished as the lesson is taught again, presumably to the same age group. In the lesson study approach, not every member of the group is involved in teaching the lesson. Relational distance might be created if a select number of teachers are perceived as "model" teachers. This approach could encourage a narrow perspective of teaching.

The steps of lesson study are similar to the process we engaged in where we studied the development of early number systems, the teachers planned individual lessons, they observed a lesson in action, and we met to discuss and reflect on the lessons. However, in my approach, each teacher designed and taught her own lesson while a colleague observed her. The observer saw a unique and personal interpretation arising from our study of number systems. She was offered the opportunity to enhance the lesson she had to subsequently teach by gleaning ideas from the observed lesson. Similarly, the teacher was given immediate feedback that framed her own observations of the next lesson. This approach recognized and celebrated the professional decisions teachers made as they constructed their own pedagogical knowledge of mathematics.

Ideas found in the literature on study groups informed my approach to professional development in this study. I was interested in dynamics of relationships and this innovation seemed to expand the learning possibilities within the community. Yet I was also interested in the history of mathematics as stories of mathematics in a human context. Possible themes for my particular study group evolved when I considered relations to *It*.

Enacting *I-Thou-It* Relations

Research on study groups and the projects of CGI, SummerMath for Teachers, and QUASAR contributed to my way of thinking about professional development. Many questions emerged when considering my own research. What was the *It* of mathematics? Did the *It* have to be a study of the history of mathematics? Why did I choose this particular *It*? Why did none of the other approaches consider this *It*? Why was a study of the history of mathematics a good *It* to offer? How could my experiences of synergy described in the Prologue be framed within these approaches to professional development? Again I return to the *I-Thou-It* framework to help me to think about these questions.

Davis (1995) explores the *It* of a Grade 6 mathematics class where students are exploring fractions. He asks, "Is the mathematics the knowledge of fractions that these students seem to be enacting—albeit on a largely unformulated level? Or is it the explorative activity? Or is it the formalized product of such activity?" (p. 3). He suggests that the *It*, often described "in terms of knowing subjects or in terms of known objects" (p.3), is really like the "subject matter of a conversation. It exists only in conversing, and its nature, its structure, and its results can never be anticipated, let alone fixed" (p. 4). If I interpret his notions within a professional development setting, the *It* could be viewed as mathematical content knowledge (the *It* of SummerMath), as the activity or process of student learning (the *It* of CGI), or as the task presented to students (the *It* of QUASAR). The *It* could be perceived as the content of conversations between participants (one possible *It* of study groups and PLCs). The various viewpoints of researchers on what *It* is makes professional development particularly challenging. *It* can be many things. I further explore the *It* of mathematics in Chapter 4 but for now I explicate why I chose the study of the history of mathematics as my particular *It* for this professional development setting.

Research findings suggest that elementary teachers improved their attitudes and changed their views of the nature of mathematics when they engaged in the study of the history of mathematics (Philippou & Christou, 1998). Liu (2003) suggests that the study of the history of mathematics promotes the view of mathematical thinking as dynamic and as a "combination of complicated processes: guessing, induction, deduction, specification, generalization, analogy, formal and informal reasoning, verification, and so on" (p. 417). This contrasts with the view of mathematics presented in textbooks and by many elementary teachers where the mathematics is a polished product. The study of the history of mathematics seemed to offer opportunities to consider philosophical questions about the human agency in its development. As discussed in Chapter 1, by focusing on historical stories, I hoped the familiarity with literature would ease teachers into the setting and would lower their resistance to viewing mathematics in an alternative way. I believed that this *It* would prompt reflections on views of mathematics as a humanity. I was interested in how mathematical experiences and images of the nature of mathematics could be altered. The study of the history of mathematics seemed to address these dimensions.

Once I had considered the *It* in the professional development setting, I contemplated the relations of *I-Thou*. Rather than fragmenting these relations into dyads, I wanted to consider how the dyadic relations were coordinated. How were these relations interconnected, interdependent, mutual, and reciprocal?

As I grappled with these questions, I began to investigate the connection between individual and social learning and considered possibilities for synergetic learning within a community. I wondered how my experiences of synergy described in the Prologue could be framed within this approach to professional development. Davis (1995) suggests that "collective knowledge and individual understanding are dynamically co-emergent phenomena" (p. 4). The notion that each teacher's knowledge was intertwined with every other teacher's led me to reconsider Buber's ideas of community. How did he view the intertwining of *I*, *Thou*, and *It*?

Buber (1966) describes a dialogical community as a third way between individualism and collectivism:

> On the far side of the subjective, on this side of the objective, on the narrow ridge where I and Thou meet, there is a realm of 'between'. This reality, whose disclosure has begun in our time, shows the way, leading beyond individualism and collectivism, for the life of future generations. Here the genuine third alternative is indicated, the knowledge of which will help to bring about the genuine person again and to establish genuine community. (p. 55)

According to Buber, a strong community relies on robust relations between *I* and *Thou* and arises out of mutual, dialogical relations. This is reminiscent of his notion of encounter, the meeting between people in relation, and reiterates his view that growth and development is only possible through living in relation to others. In other words, Buber seems to claim that individual understanding and collective knowledge occurs simultaneously. Individuals nested within dialogical communities explicate relations of *I-Thou*. Drawing on Hawkins' ideas of *I-Thou-It*, I considered possibilities for synergetic learning within professional development settings. What was the nature of the synergistic dimensions of learning that could occur and that were, in fact, present in my most profound experiences of professional development? What were the possibilities for experiences to be transformed into synergetic encounters?

Synergetic Relations

The highly relational nature of learning and social interaction is explicated by Maturana and Varela (1987):

> This circularity, this connection between action and experience, this inseparability between a particular way of being and how the world appears to us, tells us that *every act of knowing brings forth a world*. (p. 26)

If I relate the notion of action in Maturana and Varela's writing to Buber's description of encounter, it appears that Buber's notions of experience and encounter are connected rather than differentiated. The shift between *I-Thou* and *I-It* relations may tend toward recursiveness and circularity rather than suspension in the *I-It* state.

How do these ideas relate to community or social contexts of learning? In my recollection of *Synergy* (see p. 2), I encountered collective learning. In that context, we acted in a coordinated way as we communicated with each other. Our learning together was more intense and amplified than the sum of our individual ideas. I described this encounter as rich because it seemed we acted collectively. For me, this was a qualitatively unique encounter with a setting of professional learning. How can relations of encounters be prompted?

Expanding Buber's linear diagram of relations between teacher educators and teachers, adapting Hawkins' triangular model by considering relations with mathematics, and incorporating Buber's ideas of the relations among learning communities, I present my diagram for describing possible relations in professional development.

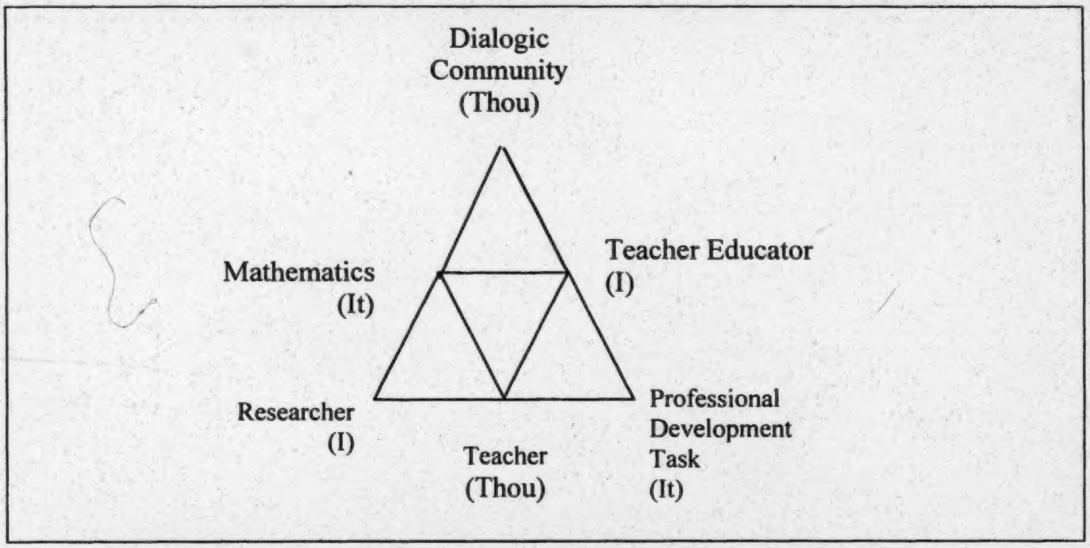

Figure 2-4. Nested relations within professional development settings

Within the context of professional development, my interpretation of Hawkins' trilogy of *I-Thou-It* is nested within a trilogy of the researcher (*I*), the dialogic community (*Thou*), and the task of professional learning (*It*). Together this represents the professional development setting, described further in Chapter 3. What I want to emphasize here is the importance of an encounter among teachers, teacher educators, and mathematics and the possibility of that encounter becoming synergetic. This diagram mirrors the self-similarity between individual relations to mathematics and collective encounters of mathematics. I believe both are essential to professional development. In fact, I claim that synergy can occur within professional development settings if and only if relations among *I*, *Thou*, and *It* are robust. Relations of encounters are necessary for rich learning. The interplay between individual encounters and synergetic communities can be investigated using this nested image.

How can individual encounters with mathematics and dialogic communities be enacted? Attending to the professional development task of my study, I chose to immerse elementary teachers in the uncertainty and indeterminacy of historical mathematical stories to see how their experiences or their images of the nature of mathematics changed.

Many elementary teachers view mathematics as isolated facts and procedures that must be memorized. The stories of their mathematical experiences limit the impact of curriculum reform in mathematics (Ball, 1990). Encouraging teachers to write an alternative story about mathematics involved reflection on (1) stories of mathematics as a discipline over time; (2) stories of the development of mathematical concepts; (3) stories of mathematicians' lives; and (4) personal stories of encounters with mathematics. My study provided and investigated sustained professional development opportunities in these four dimensions.

Working as a community to create and implement a curriculum based on a storied approach provided an entry point for teachers to experience enjoyment, to explore the nature of mathematics, and to enhance mathematical knowing. Possibilities for robust *I-Thou* relations in professional development settings point to the necessity of attending to relations with *It*. Focusing on dialogic communities and prompting strong relations of *I-Thou-It* converge to expand learning possibilities for teachers of elementary mathematics.

What possible tasks might encourage strong relations to *It*? How might synergetic relations be prompted? How could relations of *I-Thou-It* be enacted as more than an affair of mirrors? My responses to these questions are presented in Chapter 3.

Chapter 3 – Beckoning to the Community

An *I*, a *Thou*, and an *It*

> *I glanced around the classroom for a chair. Finding one in the corner, I pulled it up to the teacher's desk and began to explain my study: "I am interested in working with some teachers to design a curriculum based on the history of mathematics." The teacher looked up from his computer where he was entering marks for report cards. "I have no time for you people from the ivory tower!" I sat forward and hastened to explain that I was a teacher. He was not convinced and stated that he would give me five minutes of his time only because the principal had made this appointment for us. He continued to work on the computer while I described how stories could be used in mathematics classes. After exactly five minutes, I was ushered out of his classroom. I muttered angrily as I marched back to my car. I was an* It.
>
> *(Personal reflection, March, 2004)*

Throughout the previous chapters, I described different approaches to professional development. Attending to Buber's relations of *I-Thou* and *I-It* and Hawkins' relations of *I-Thou-It*, I was aware that many of these approaches portrayed a variety of possible *I-Thou* interactions. These included a focus on individual learning through transmission of material by an expert, on collaborative learning facilitated by a guide, and on group learning through interacting and collaborating with members of a community.

Similarly, many of these innovations focused on different forms of *It*. For example, *It* involved a range of tasks that included engaging with mathematical content as a learner, analyzing student thinking, and discussing cases of pedagogical practice. My dissertation is a study of one particular *It*.

In order to investigate professional development as a dialogic encounter, I considered those aspects of the systemized approaches that resonated with my own experiences and chose to emphasize common features of these initiatives that seemed to promote teacher learning. I focused on providing opportunities for collaborating in a shared endeavour, interacting with other teachers from diverse backgrounds, creating instructional tasks, observing the classrooms of their colleagues, reflecting on their own personal experiences, working on mathematics, and influencing the agenda of our meetings.

This chapter describes the methodology of my study and is organized into three sections. The first section focuses on *I-Thou* relations and describes my process of finding participants. Here I consider conditions of a dialogic community such as collaboration and diversity. In section two, I explicate relations of *Thou-It*, where *Thou* is the dialogic community and *It* represents the professional development task we engaged in. I describe how I attended to the tasks of the study group. Section three seeks to connect relations of *I* as a researcher to the *It* of the professional development setting. Here I describe my methods of data collection and analysis within a context of synergetic interactions.

Relations of *I-Thou*: Finding Participants

Finding participants for my study was very difficult. In fact, this experience filled me with angst. A member of my candidacy examining committee was very concerned about the pragmatic details of my research but I was naïve. By ignoring her apprehension, I was unprepared for the obstacles I encountered.

In order to attend to diversity, I decided to invite three teachers to participate in the research. I thought that this number would be small enough to encourage active engagement and large enough to ensure some variety. In addition, if one member withdrew from the study, our learning system would be able to adapt to this change. Inviting three participants allowed me to balance these concerns.

To ensure greater diversity, I did not focus on one grade level but sent letters to intermediate teachers. I also included teachers from different schools whose different teaching contexts would contribute to the richness of our encounters. I considered prior experiences with mathematics and chose teachers who ranged from reluctant problem solvers to confident mathematicians. I felt that collaboration would be implicitly addressed because I was inviting teachers to participate in a specific endeavour and I inferred that people with a common interest in the history of mathematics would participate. Also, I limited participation to teachers working full-time with a homeroom class of students. This addressed the features of collaboration and diversity.

When inviting teachers to participate, I did not have a wide response to my letters of introduction. Finding participants involved a hierarchical process of contacting the superintendent of the school district for permission to approach the principals of individual schools. I left letters of invitation with the principals to forward to their teachers. Some principals distributed my letter, others decided that their schools were too involved in mandated professional development initiatives and declined my request. In the instances where teachers were given my letters, they were asked to contact me for more information. On one rare occasion, the principal arranged meetings between his staff members and myself. The memory described in *An I, a Thou, and an It* (see p. 40) shows the futility of this procedure.

During a six-month period, I had seven responses. One teacher decided she was not able to participate because of time commitments to her children. Another teacher was not included in the study because I felt she had her own agenda for participating. Since it had taken a long period of time to find participants, I decided to include all remaining teachers who were interested in participating. Thus, I began the study with five teachers. (see Appendix A for my story of this process).

All teacher participants worked at different schools within a common geographical region. These teachers were situated in two school districts: public and private. Three teachers worked in a small city and the other two worked in rural community schools. We decided to alternate our meetings between the teachers' classrooms of the city schools in order to reduce travel time.

Three of the individual teachers taught for the public school board system. This board and its teachers were embroiled in a labour dispute. During the study, the teachers voted overwhelmingly to strike and the board threatened to lock them out of the classrooms. The labour tensions persisted throughout the study, however no action was taken. Despite this unrest, the teachers remained committed to the project. Throughout the study, I was apprehensive about the situation and this may well have contributed to my concern about the agendas of our meetings. Although I was aware that professional development that results in qualitative change in a teacher's thinking and pedagogy is an autonomous activity and that externally imposed agendas are doomed to failure because they are simplistic solutions to the complex endeavour of professional learning (Castle & Aichele, 1994), I grappled with this tension throughout the study (see Interludes 2 and 3).

The process of finding participants immersed me in the practicalities of engaging in qualitative research. During the six months it took me to find participants, I came to understand the importance of relationships and the time required to build these relationships. Certainly I have a deeper understanding of the importance of relational responsibility as I interact with teachers who may become participants in future research studies. The process of finding participants provided me with many opportunities to reflect on my role as a teacher educator.

I started the project with five teachers but funding cuts eliminated the teaching position of one of the participants who consequently opted to withdraw from the study. As we continued to engage in learning, our relationships evolved. Although these teachers became known more fully throughout the study, I offer a brief introduction reflective of how they introduced themselves to me.

Martha

Martha had been teaching a variety of full-time, part-time, school-based, and alternative programs for 15 years. She had lived in a variety of communities. She was a mom to two young children. Her strong commitment to professional development was evident in her participation as a graduate student (she had completed a Master of Education degree and a diploma in adult education) and as a school leader in integrating technology into the curriculum. At the time of the study, she was a school technology leader who initiated teacher-developed projects, assisted teachers in their classrooms, engaged in training workshops, and led the staff at her school through the training program.

Martha described herself as getting better at being a mathematician. Her interest in this project was because she wanted to have "one more trick in the bag of tricks" and because "obviously it's another way." She believed that "the more connections that we can make for kids in making things fit together, I think the better they're going to do all around" and she hoped this would result in increased achievement of her students as the math made more sense to them:

> It will help me understand more about how the kids understand math, what makes sense to them. Because I've only got what makes sense to me to draw on. So if we can increase the ways, the number of paths to get there, then it's got to be better for somebody.

Martha taught a combined class of grades 3 and 4 students.

Barb

Like Martha, Barb had been teaching full-time and part-time in various grade levels in various communities. Her school career spanned approximately 25 years and included a year of teaching in Australia in a teacher exchange program. Her children were grown, she had lost her first husband, and had recently remarried. She demonstrated a strong commitment to professional development as she participated as a leader in a school-based learning community initiative. She was involved as a staff representative for the teachers' association and had previous experiences as a participant in university research studies.

Barb described herself as "an adequate mathematician. I enjoy math and I thoroughly enjoy teaching it. I love to teach it." She wanted to be involved in this study for two reasons:

> One, it's important for children to make connections. And two, I believe children have a true curiosity of where does this come from. And they want to know why. As you know a two year old and a three year old, one of the first questions is why? Why? Why? And it's not just that they've learned that question. I truly believe that if it has some relevance to them, as with all of us, we are more interested in learning it. And so, at this age, they love stories. And they're very interested in stories. And of course I would tell them that history is a story. And mathwise, they love to hear stories and I think that they would remember those concepts better if there are stories attached to it. Some children have, and I don't know whether it's visual or what kind of learning (it would be good to do some more research on the brain, how it sees things), but if they just see numbers, they have a real aversion to that. So we have to approach mathematics from many different ways in order to help students learn and in order to help me meet the needs of all my different types of learners. I think this is one way. You can reach a child who has more language arts strength, by teaching mathematical concepts through story.

She believed that her students would benefit from this experience:

> They'll get another way of looking at math which I'm hoping will appeal to those students who are more inclined in language arts. So it will give them another in to look at math in a different way. Hopefully help them to improve their marks, but not just their marks, but just a different way of looking at mathematics rather than just algorithms and numbers.

She felt that there would be many benefits to her:

> I'll learn a lot more about history of math, which I'm interested in as well. So it will give me another avenue to help my students, to help teach another way. The more ways I have, the more I can reach those students that need that.

Barb taught grade 5.

Christine

Christine had recently moved from another province where she taught for 14 years. Having been in this area for 2 years, she was interested in getting to know other teachers in the community. She was married and did not have any children. She had attended several workshops on various subjects but had not assumed a leadership role in professional development.

Christine expressed her willingness to learn about mathematics even though this was a subject that she was not comfortable teaching. In describing her background in mathematics, she told me:

> I never, ever enjoyed it because I am only kind of getting to enjoy it the longer I teach it. So if I can find a way to make it easy or make it fun or make it, 'Oh, so

> this isn't such a big deal after all' then that's a good thing because then I enjoy it, too. If it's combined with something, I find it much more interesting.

She was interested in this project:

> I could see that it would benefit my abilities to teach math and relate to those things and interconnect them. I could see strong connections with social, strong connections with language and stuff like that. But I'm not that confident in putting them together and working with them. So I can see that as being an advantage, my own confidence increasing. And I like the idea of working with other people, talking to other people in the system, and just getting an idea of how are people doing this? That's exciting because I haven't had contact with very many teachers outside of this school and the [other] one school I was at since I've been here. It seems really small to me. It's hard to make those contacts when you're doing the job all the time.

She wanted her students to be excited about this project so that they would talk to other students about it:

> It'd be exciting to see them pass it on to others. And effectively. Not just pass it on because they figure it works but pass it on because they know why. Because then they're teaching each other in a sense. And that would be really neat.

Christine taught grade 4.

Denise

Denise taught for 16 years, all in this school division. She was married and had two teen-aged boys. She was a strong leader in professional development, having participated as a school mentor to new teachers and as a lead teacher with a district-initiated project in mathematics assessment. She was working on a diploma in music education through the university and was a lead teacher in the implementation of technology. Her commitment to learning was evident in how she spent her holidays. "Every summer I like to do something new or learn something new, whether it's to juggle or play a new card game."

In relation to mathematics, Denise described herself as "a lifelong learner." She did not enjoy mathematics in school, having experienced some frustration. As a teacher, she commented:

> I feel comfortable with math but I can't say I feel confident with math either. I always wonder if I'm doing a good job or if I need to repeat something or 'why isn't he getting it?' Is it me, is it something that I'm not saying properly or doing properly? Comfortable but not confident.

She was interested in this project:

> I'm hoping to increase my skills and my comfort level with math and my whole idea of 'Am I on the right track?' kind of thing. I like the idea of having the fresh new ideas from the university or from you because I think it's easy to get stagnant. I'm always one to try to keep up my PD. If we're given a certain amount of money, I use it every year on something because there's nothing worse than a teacher that just stays in her own school, in her own classroom, and does the same thing year after year. I just want to do the best for the students and I want it to be comfortable for me where I actually feel competent, where I

> delivered something and felt that they got it, and that everybody achieved good grades and things in their math assignments and just come away feeling really comfortable with what they're doing, feel like they're capable and that they can go on.

One aspect of this project that she found appealing was the opportunity to visit other teachers' classrooms. Denise taught grade 3.

Although I had many difficulties finding participants, the self-selection process likely contributed to the common learning interests of these teachers. In addition, the setting within the teaching community linked these teachers and provided a common context for our study.

Relations of *I-Thou-It*: Designing the Study

As a community, our relationships developed in the setting of a study group. Because I wanted to investigate participant interpretations of our encounter as individuals as well as members of the community, I arranged to talk with each teacher at the beginning and midway through our agreed-upon time allotment. Personal reflections arising within these individual conversations were willingly shared during group meetings.

In the initial part of my research, it was important to provide an opportunity for participants to appreciate their common experiences and expectations. Thus, the beginning conversation explored each teacher's perspective on the nature of mathematics, their background experiences, and their familiarity with the content of mathematics. After an initial meeting as a group in spring, we continued to meet bi-weekly throughout the fall and winter months. Each of these meetings lasted between 75 and 90 minutes. At the beginning of the project, we had agreed to meet until the end of December. This would have consisted of 11 meetings. However, the project was extended because the teachers wanted to continue their engagement in the study (see Interlude 3). Individual conversations in January investigated each participant's impression of her own learning.

If I consider the setting of our dialogic community as a classroom, this allows me to describe our *I-Thou* relations in terms of the pedagogy of professional development and our relations to *It* in terms of the curriculum of professional development. The classroom setting of the dialogic community is described next.

The Pedagogy of Professional Development

Ideas from the literature on study groups (see p. 32) informed my pedagogy as a teacher educator. I was committed to the notion of synergetic collaboration as I strove to encourage what I now see as strong *I-Thou* relationships. I participated in each of the tasks and resisted offering an expert opinion since I believed it was necessary for the emergence of a learning community for our group to become mutually dialogical.

For me, the notion of 'expert' was problematic. Sometimes this was a position people got put into. Sometimes they placed themselves in this role. I had experienced both contexts, as shown in my recollections in the Prologue. In many of these contexts, my notion of expert was linked to an authoritative and dominant figure. I realize now that this view limited my relationship with the teachers as I hesitated to share my mathematical expertise. What role do experts play in a learning community?

Buber claims that leaders live a dialogical life. They express, represent, and animate the *I-Thou* relation. Describing the relationship between the teacher and the students, Buber (1947) states that "the relation in education is one of pure dialogue" (p. 125). Drawing on this notion, relationships within a professional development setting must be dialogic.

Reframing my ideas of expert and expertise, I now understand that some people have a stronger influence on the agenda of the group because of their experience. It is interesting to note that the terms expert and experience share the same root, *exper*. After our first meeting in fall, I wondered about my role as a participant. I felt that Barb had established herself as a leader of the group and I needed to be sure that my agenda was met. In my field notes, I wrote that I wanted to negotiate this within the context of our discussions. Barb's teaching experience contributed to a richer conversation about teaching and learning mathematics. I saw her as a leader but did not identify her as an expert. Reflecting on this, I realize she was bringing her expertise to our group in a dialogical way. Returning to the notion of collaborating in a shared endeavour, I wanted our interactions to be authored by group members, not authorized by me. In many ways, I deliberately withdrew my expertise to avoid being perceived as an authoritative figure. Perhaps through this withdrawal, my dialogical interactions were reduced.

What I seemed to be avoiding were relations of distance. In many of my previous experiences of professional development, the focus was on relations of *Thou-It* and the *I* floated as a facilitator. Two legs of the triangle were missing or faint and I believed this limited possibilities for professional learning. In the figure below, I present the situation I was trying to avoid.

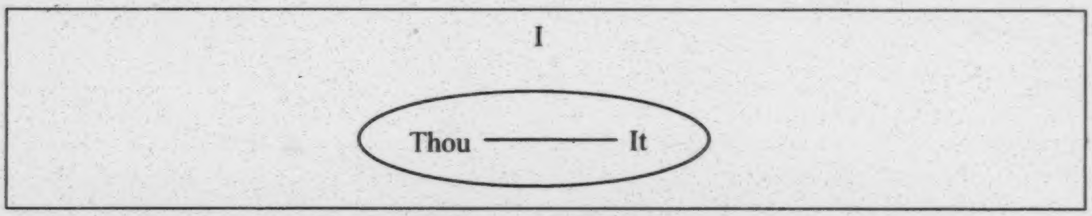

Figure 3-5. Possible relations of *I-Thou-It* in professional development settings

Why was the *I-Thou* relationship so important when my goal was to try to strengthen the relationship between the teacher and the mathematics? Buber's writings suggest that a dialogic community depends on strong *I-Thou* relations. Hawkins draws attention to the necessity of a triad of *I-Thou-It* for learning within a classroom setting. Perhaps for me, my relationship to the teachers was important because I wanted to participate in the inquiry. I felt that this interactivity would benefit me as a researcher by allowing me to refine and recast my conceptual frameworks as I was confronted with the realities and constraints of the classroom context (Huberman, 1999). My desire to bridge the perceived gap between research and practice was strong. By working in relation with these teachers, I hoped to participate as an equal member in the learning community. I have come to understand that this notion of equality involves differentiation of expertise but the tension I experienced as a full participant remained throughout the study. I discuss the challenges I encountered while resisting a leadership role in Interlude 2.

Castle & Aichele (1994) suggest that learning occurs when teachers choose to be involved. Similarly, the importance of teacher autonomy and encouraging teacher choice is evident in studies of the professional development of mathematics teachers (Ball, 1996; Clarke, 1994). For me, it was important to enter the relationship by respecting the teachers' professional knowledge and experiences.

To demonstrate my respect for teachers I provided release time. Arbaugh (2003) found that teachers in her study group appreciated release time for meeting and my own experiences in professional development settings are consistent with this finding. Since this was not possible for my study because of limited resources, I decided to teach in each classroom once a week. Teachers decided how they would spend this time. Some remained in the classroom while I taught, others worked with individual students while I worked with the rest of the class, still others used the time as a preparation period. This provided an opportunity for me to strengthen *I-Thou* relations as I worked alongside each teacher.

In addition to the weekly lessons, we arranged three rounds of classroom visits. At the end of the three cycles, each teacher had visited each of the other teachers' classrooms for an afternoon or morning. My weekly lessons with the students provided consistency when I released teachers for half-days during these classroom observation exchanges. By teaching in each classroom every week and by having teachers visit each classroom, the students became part of the professional development setting. We were able to share our interpretations of our interactions with students and these resonated because we were familiar with both the students and the school context.

The literature on study groups suggests that collaborative learning is important. One of the principles for effective professional development that Clarke (1994) proposes is working in groups. Little (1993) suggests that effective professional development offers embodied engagement with ideas, materials, and colleagues. In designing my study, I wanted to provide opportunities for both individual and social learning. Working as a study group seemed to offer this possibility.

Our meetings became places of mutual dialogue as our stilted monologues became rich dialogues. Because I was part of the community, I engaged in conversations with other members. A conversational approach was intentionally chosen. Gordon Calvert (2001) describes genuine mathematical conversations as occurring when people attempt to reach an understanding of some mathematical phenomenon. The relationship between people is emphasized as she writes, "Conversation carries with it a sense of embodiment, presence, responsiveness, and responsibility. In our conversations with others we explore a topic of mutual interest and at the same time further our relationship with each other" (p. 47). The tenor of a conversation complemented the purpose of my research. Moreover, since I was interested in emerging teacher learning, I wanted to acknowledge the dynamic nature of mathematics. Gordon Calvert comments:

> Doing mathematics is often perceived as independent, linear and goal-directed behaviour, while a conversation rarely proceeds in a linear progressive manner with clear direction and purpose. Its character is more circular, weaving its way around and through the topic at hand. (p. 47)

A conversation is dialogic, mutual, and complex. This pedagogical dimension was important to me as I attempted to foster strong *I-Thou* relations.

Dialogical interactions were prompted wherever possible. I attempted to offer support for informed disagreement as Little (1993) suggests. One site for this was through the sharing of personal stories. As part of this study, teachers were encouraged to reflect on their own stories of mathematical experiences and encounters and on their evolving notions of the nature of mathematics. This took the form of journal writing and group conversations.

Carter (1993) writes, "To understand thinking, then, it is necessary to find the story that structures an individual's model or theory of events" (p. 7). Like her, I believe the richness of human experience can be expressed best through story and story allows us to interpret and comprehend our experiences. Similarly, Davis (1996) links experiential understanding to storying:

> Formulating one's understandings is regarded as a process analogous to storying, whereby the learner incorporates new experiences into the text of previous understandings. In this process, both past knowings and interpretations of current experiences are revised—one rewriting the other, as it were. This revisionary and recursive storying process stands in stark contrast to the cumulative and linear storying process of transmission-acquisition models of learning (especially those founded on "mind as computer" metaphors), thus highlighting not just the reproductive nature of understanding but its productive aspects as well. Understanding is generative—both of the selves that participate in the understanding and of the world that is opened (or "brought forth") by that understanding. (p. 204)

Through narrative interpretation, stories can be retold. Clandinin and Connelly (2000) claim that "narrative is the best way of representing and understanding experience" (p. 18). The retelling and reshaping of stories helps change our understanding of the world. I included dimensions of narrative inquiry as one way to prompt the sharing of, and reflection on, stories within the group.

The Curriculum of Professional Development

I began the project by meeting individually with each teacher. I was interested in attending to the experiences of teachers and their prior beliefs. This provided an initial meeting of *I-Thou* within an intimate context. Investigating experiences and beliefs of teachers is proposed as a principle of professional development by Little (1993), Castle and Aichele (1994), and Ball (1996). These initial conversations prompted further interactions within our group meetings as we shared our thoughts about the nature of mathematics, our experiences about teaching mathematics, our interest in studying the history of mathematics, and our notions about the integration of literature and mathematics.

The curriculum of my study involved offering rich professional development tasks to encourage dialogical interactions. Using ideas from study group approaches, I designed tasks that offered an opportunity for individual and collective action as participants encounter mathematical ideas. These tasks were *proscriptive* rather than *prescriptive* in the sense that we had parameters for our study but we were able to craft our story and lessons in creative ways within these limitations. This reflects a notion of proscription presented by Davis and Simmt (2003). They explain that prescribed tasks are based on predetermined outcomes and agendas to achieve the outcomes. In other words, *prescribed* tasks specify what is allowed and everything else is forbidden. In contrast, *proscribed* tasks specify what is forbidden. These boundaries and conditions constrain the activity but enlarge the possibilities because everything else is allowed.

During the first phase of the study, we collectively identified the curricular area of number concepts as our focus. We worked closely together as we gathered resources, wrote stories, planned student tasks, and created lesson plans. To begin our investigation, I offered selected readings on the history of numbers and we shared our interpretation of this literature. As research indicates, effective professional development approaches consider participants' subject-matter learning (Cooney, 1994; Hyde, Ormiston, & Hyde, 1994; NCTM, 1991). These readings prompted our investigations of related mathematics and included an opportunity for teachers to increase their content knowledge of mathematics as they situated the mathematics within a historical context.

I chose the term *anecdote* to describe the type of story to be created. An anecdote can be defined as a short entertaining story about a real incident or person (Pearsall, 2001). The emphasis on event or character was central to my focus on the humanity of mathematics. The brevity of the story implies that an anecdote is part of a larger context. Therefore, our anecdotes would not represent a full story of mathematics but would be told because they made a relevant point. Throughout the study we investigated this description and redefined our anecdote as a folktale, a story of historical fiction, and a parable (see Chapters 6 and 7).

The entertainment aspect of an anecdote implies the acceptance of a certain sense of poetic license. Derived from the Greek word *anekdota* meaning 'things unpublished', anecdotes could include details not published. Thus, anecdotes have a quality of hearsay and I liked how this matched the storied nature of mathematics. Haury (2001) writes, "Some of us may feel uncomfortable with the notion that mathematics is fiction, but the concepts and procedures of mathematics are all constructions of our minds, products of our attempts to understand our worlds, real and imaginary" (p. 3). In admitting this quality of story, I prompted conversations on the myth of rendering a factual and "true" account of history.

While we investigated ideas for our stories, we talked about issues arising from our work. During the first month, we reflected on many questions: How were numbers developed? How was mathematics like a story? During the second month, we each wrote an anecdote about numbers and shared these with the group. I had anticipated that we would somehow sequence or categorize the stories and work with all of them, but we selected one as it seemed to resonate with each of us. We were able to see the potential of this story and decided to abandon the others. I investigate this phenomenon in Chapter 6.

As we rewrote and refined our anecdote, we investigated issues arising from our work. What did this story tell our students about the nature of mathematics? What kind of story was this? What was the role of story problems within the context of story writing? Why might this be called a folktale? To what extent was this a story about mathematics? Our conversations were focused on our story and this gave a context for revisions. As we became satisfied with our anecdote, we brainstormed possible student tasks. These conversations were rich as we reflected on the purpose for the tasks and connected these to student learning. The teachers decided that they would each like to select a task to try in their classroom. I distributed a list of our ideas and left the lesson planning to each individual teacher. Thus, we did not spend time in our meetings to collectively design a lesson plan. By the middle of November, we were ready to teach our lessons. At the mid-point in this phase, the mother of one of the participants volunteered to illustrate our story. We decided to create a book and include suggestions for teachers. This sparked much enthusiasm and synergy. I explicate this further in Chapters 7 and 8.

The second phase of our study focused on the implementation of the lessons. During this phase, classrooms became nested within our community and became sites for dialogical interactions. Throughout this phase I provided release time for teachers so they could visit another member's classroom. Each teacher was paired with another teacher. While one person was teaching, the other made observations. I suggested that they make notes that described the lesson and its effect on students. They were asked to gather evidence of students' mathematical thinking. In this structure, teachers had the opportunity to become observers and record research notes. Conversations about the practice of teaching and learning occurred because we engaged in classroom visits. These conversations became sites for mutual interactions in that teachers were able to envision other ways of interpreting the story. An impressionable aspect of this experience was that teachers were able to interact with the students in a role other than a teacher. It would have been fruitful to engage in some tasks from the CGI project. However, the scope of this project limited our focus on student learning. For the purposes of this study, we decided to write field notes as the teachers observed the lesson being taught and to gather student artefacts from the lessons. The notes that they made became a source of reflection and critique during subsequent meetings. We decided not to focus on an analysis of student learning. Rather our conversations emphasized our impressions of their learning.

In the third phase, we used a presentation software program to format the story and series of lessons. We reworked each lesson so that it could be included on a compact disk. Each teacher wrote a reflection of their lesson plan and commented on its implementation in the classroom. This allowed them to show explicitly the connections they were making between their lessons and their experiences of mathematics. It became an artefact of the learning within our study group. This technology was well suited to my research as I prompted reflection on the stories of teachers' encounters with mathematics. Specifically, I was able to investigate the teachers' experiences by documenting the connections they made to the mathematics. Working with the presentation software program engaged us in reflective thought and critical pedagogy.

In January, I met with each individual teacher. These conversations provided an opportunity for me to investigate the impact of participating in this professional development setting from the teachers' perspectives. Within the community, several teachers seem hesitant to contribute and time limited the amount each person could share. I decided that I wanted each teacher to have an opportunity to reflect on her experiences of learning within our group. I believed that this would encourage stronger *I-Thou* relations and consequently, stronger *I-Thou-It* relations within the group. I met with each teacher for about 75 minutes and revisited many of the topics that had emerged in our group conversations. We talked about the nature of mathematics and how they saw themselves as mathematicians. I asked them to share what they had learned about the history of mathematics and the role of story in mathematics. They shared tensions and joys that they had experienced during the previous months and many articulated their views of the long-term influence of this experience on their teaching. Each teacher seemed comfortable with the conversation and continued to talk with me after I had turned off the recorder.

After these individual conversations, we continued to meet once a month as a group to complete the website. We were focused on finishing the project and worked together to refine the presentation of our lessons and to select student artefacts for inclusion. Teacher comments and reflections were written during this time but these were focused on what they thought might be helpful to other teachers. We concluded with a celebration supper (see Appendix B for a timeline of the meetings).

Nested Relations of *I-Thou-It*: Analysing the Study

While we were continuing to meet once a month during the spring, I began to write my interpretation of our encounters. Gadamer (1989) recognizes that every act of research is an act of interpretation. Returning to Buber's relations of *I-Thou* and *I-It*, I was reminded that this stage of my research threatened to objectify our interactions and shift teachers to a relation of *I-It*. Here I was a researcher and the teachers were my objects of study. My interactions with them were limited. As I remembered our encounters, I sought to interpret our relations with compassion. This analysis relied on my active and generative participation in the interpretive process as I attempted to do trustworthy research while writing in relation of *I-Thou-It* nested within our professional development setting.

When describing non-linear systems of chaos, Gleick (1987) offers a metaphor: "The nature of those relationships often becomes impossible to pin down. Analyzing…is like walking through a maze whose walls rearrange themselves with each step you take" (p. 24). For me, this portrayed my experience of the interpretive process of my research. My organizational structures and categories were dynamic and responsive to the unfolding nature of our conversations. They were constantly rearranging themselves and the process was recursive as I tried to make sense of the data. What is offered here is one interpretation of how the walls were in alignment in one particular moment in time. However, I realize that the walls continue to shift.

Bi-weekly teacher reflections were written by all participants and collected during the creation and implementation of this series of lessons. Some of the questions and comments that prompted our reflections came from previous conversations. Themes arose from these conversations and included metaphors of mathematics, the role of language in mathematical understanding, the development of numbers, connections between mathematics and story, and the implicit messages about the nature of mathematics in the stories we created. Sometimes I initiated the conversation, often I chose emerging themes from previous conversations for further investigation. Many times the conversation was directly connected to problems encountered by teachers in the classroom. Although these were not part of my 'agenda', they became places where we could investigate our notions of the nature of mathematics and they provided occasions for prompting dialogical interactions. Collective and individual conversations were recorded and transcribed. Curriculum artefacts and observations of students were gathered. I wrote detailed field notes after each group meeting and after each day of teaching while providing release time.

My analysis involved hermeneutic questioning of the data. The hermeneutic circle, comprised of a forward arc and a backward arc, describes my research process (Heidegger, 1996). In this study, I considered the experiences of the collective and the experiences of each individual nested within this learning community. The forward arc, projection, investigated my pre-understandings of my philosophical, epistemological, and ontological frameworks. These included my relationship to the research questions and were embedded in my history. My writings in the Prologue and Chapter 1 portrayed these pre-understandings.

Gadamer presents the need for us to remain open to each other's meaning as we continue to question our pre-understandings. It is this questioning that formed the hermeneutic task. Our openness to others "always includes our situating the other meaning in relation to the whole of our own meanings or ourselves in relation to it" (p. 268). In many ways, this reflects Buber's relation of *I-Thou*. Because I was interested in our mutual experiences, the forward arc involved forming first impressions and making sense of the context by considering my pre-understandings. Members of the community were invited to participate in this interpretive process during the initial and bi-weekly conversations as themes were identified and investigated. Three themes emerged: mathematics is a language, mathematics is a story, and mathematics is a history.

The backward arc, evaluation, sought to uncover contradictions, omissions, and confirmations of my initial interpretation. During this stage, I systematically examined the data in terms of coherence with my initial interpretation of the three themes. I analyzed each of the themes by considering sites of synergetic interactions. I attended specifically to the conversational discourses and identified places where the interactions offered expanded possibilities and insights within the community. My analysis was informed by the studies of interaction patterns researched by Bauersfeld (1992), Cobb (1999), Sfard and Kieran (2001), and Zack and Graves (2001) and studies of the situative perspective in cognitive science researched by Putnam and Borko (2000). Since our interactions were discourse-based, I looked for patterns where the talk was exploratory (Crespo, 2002). These were places where it appeared that teachers were seeking and showing intellectual involvement, where there was evidence of disagreement or confusion because of dialogical interactions, or where the talk was generative and collaborative. In other words, I attempted to identify places in our conversations where robust *I-Thou-It* relations within each theme appeared to emerge.

While I initially expressed these themes as metaphors in order to describe the categories, I soon realized that this metaphoric language was conceptual as well as linguistic. Pimm (1987) claims that "metaphor is as central to the expression of mathematical meaning, as it is to the expression of meaning in everyday language" (pp. 10-11). The teachers in my study used metaphors to express how they were making sense of their experiences with mathematics and I prompted dialogical interactions by introducing new metaphors of mathematics. I believed that explicating their metaphors of mathematics offered a means for understanding their images of mathematics. I investigate the significance of metaphors and mathematical understanding further in Chapter 4.

The research questions were reframed throughout this process and my writing proceeded in a recursive manner as I reinterpreted my data within a framework of relations. Entire chapters were rewritten and reframed. When evaluating my interpretive account, I considered the extent to which the research concerns are addressed. Three research questions emerged to frame this study. (1) What are teachers' individual and collective encounters of mathematics when creating and implementing a series of lessons based on a historical story for elementary mathematics classrooms? (2) What are my experiences of working on the mathematics as a teacher educator? and (3) How do we, as members of a learning community, experience our relationship with each other? Each of these questions was investigated within the three dyadic relations of *Thou-It* (Chapters 4 – 8), relations of *I-It* (Interludes 1 – 3), and relations of *I-Thou* (Chapter 9) that run throughout the rest of this account.

I chose to write in a style that is both narrative and phenomenological. In the initial chapters, I crafted recollections or memories as rich descriptions of my experiences These recollections or memories, common rhetorical devices of phenomenological writing, were constructed from the original data and are indicated by italicized blocked text. At the beginning of the rest of the chapters, I provide excerpts of conversations where I believe synergetic relations were evident. I have tried to preserve the voices of my participants as I interpret our encounters. A narrative style of writing was a good fit with my focus on *mathematics is story*.

I am grateful and honoured to have learned alongside such dedicated, enthusiastic, and compassionate teachers. Recursively returning to a relation of *I-Thou*, I offer my interpretation of *I-Thou-It* relationships within this particular professional development setting.

Interlude 1 – Working on Relations of *I-Thou* through attention to *It*

Meeting *Thou*[10]

> *When we walk our way and encounter a man who comes toward us, walking his way, we know our way only and not his; for his comes to life for us only in the encounter. Of the perfect relational process we know in the manner of having lived through it our going forth, our way. The other part merely happens to us, we do not know it. It happens to us in the encounter. […] The You confronts me. But I enter into a direct relationship to it. Thus the relationship is at once being chosen and choosing, passive and active.*
>
> *(Buber, pp. 124-125)*

Buber writes extensively about the primacy of relations. Based on my own experiences of teacher learning, I was beginning to understand the importance of relations within professional development settings. In order to meet my teachers and to begin *I-Thou* relations, I designed the study so that I could encounter them in their classrooms. As I met the teachers, I wanted to know their ways. Thus, a significant dimension of this research involved my provision of release time for the teachers. I decided to supply this personally rather than to hire substitute teachers. This offered a unique opportunity for me to interact intensely with each teacher and her students.

I organized the release time in two ways. For the duration of the project, I presented a weekly mathematics lesson involving a series of stories and associated problems to the students of each teacher. These lessons were scheduled as part of the students' regular program. During this time, teachers could choose if they wanted to remain in the classroom. In addition, I taught three blocks of half-days to each group of students as teachers participated in classroom visits. Consequently, I had many opportunities to interact with the teachers and the students because of my frequent presence in their classrooms.

Clearly, the main reason for structuring the release time in this way was for me to develop strong *I-Thou* relations with each teacher. I believed this would provide tangible evidence to the teachers that I valued their time. Since I saw them every week, this provided an opportunity to engage in consistent, ongoing conversations about mathematics. Developing *I-Thou* relations with the students was also desired. I wanted the students to feel comfortable with me during the half-day release time and I thought this would provide the necessary familiarity. Students and teachers alike were at ease with my presence in the classroom by the time the teacher visits took place. Providing release time in this manner helped initiate strong *I-Thou* relations.

Another reason for providing release time was to augment the students' experiences of mathematics to include alternative problem-solving contexts. In addition to providing an opportunity for what I considered to be a rich learning encounter, I believed my lessons would familiarize them with the format of the teacher-created story and lesson. Moreover, I hoped to develop my own thinking about how historical stories could be implemented in the classroom by engaging with the students. What did I consider important when presenting historical stories to the students? What criteria did I use when creating my stories?

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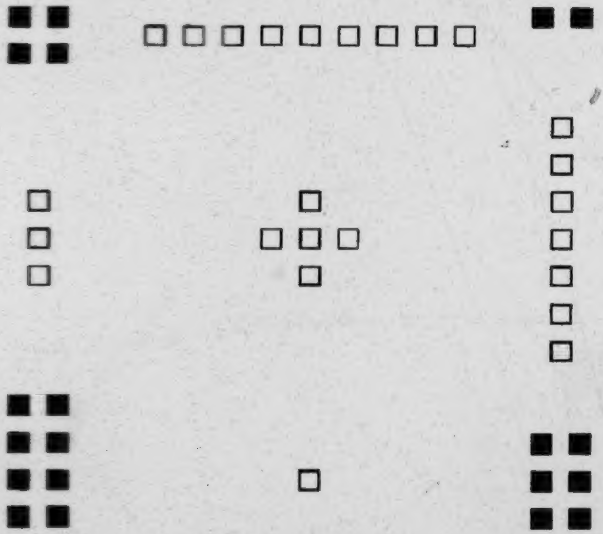
[10] Smith and Kaufmann, the two main translators of *I and Thou*, both use the terms *meeting* and *encounter* interchangeably. My use of these terms in this chapter is consistent with this.

What is *It* for me?

> *Long, long ago in ancient China there lived an emperor who wanted to lead his people in a good way. His country was poor and his father had not been a very good ruler. One day when he was sitting in his pagoda, a turtle crawled up from the Yellow River. On the turtle's back was a puzzle*[11]*:*



> *The emperor knew that if he could solve the problem, he would have an answer to how he should govern. He assembled the wise people of the land to help him with the problem. Part of the group included a young girl. She looked at the puzzle and thought hard about it. She went home and thought hard. She had supper and thought about it. She went to bed and woke up and had breakfast and thought about it. Finally, she figured out what the puzzle meant.*
>
> *When the emperor heard the answer, he knew that numbers could be used for solving problems. He decided to use numbers to measure out parcels of land so that the farmers could produce adequate amounts of food from the crops. The people of the country did not go hungry again.*
>
> *(Lesson, September 22)*

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[11] Adapted from Smith (2001), p. 97.

The classic mathematical story of magic squares presented above was the first story I told to the students[12]. I crafted the tale by relying on accounts recorded in books on the history of mathematics and by embellishing the character descriptions and story details. Based on an ancient Chinese legend, I chose to place the puzzle at the story's centre; the mathematics was explicitly embedded in it. The story was not completed until the hearer had engaged in solving the problem. By beginning with the mathematical problem and connecting it to its history by fleshing out the details, I attempted to introduce students to a human dimension of mathematics while engaging them in a mathematical task.

My criteria for creating a historical story included a strong focus on the mathematical task, in this case, the task of figuring out what the pattern of black and white squares represented. It was important to me that the task be embedded in the story. In other words, the students did not hear the entire story until they had worked on the mathematical task. Another criterion for the story was its appeal to the students. When writing the story, I wanted to add enough details that resonated with the students so that their interest in historical problems solved by humans was stimulated. Often I included a female character who was mathematically successful and a few of the stories were about children and the problems they encountered and solved.

What was I learning about the implementation of historical stories in elementary mathematics classrooms? What tensions did I experience?

Teaching with historical stories was more difficult than I had envisioned. For me, the tension between the narrative aspects of the story and the paradigmatic nature of the puzzle was strong[13]. I experienced unease because many students became preoccupied by the setting and wondered what a pagoda looked like. The narrative aspects of the story lingered in the students' work and several groups suggested that the centre part of the diagram was a pagoda. In order to refocus the students' attention on the mathematics, I had to take an active role as a teacher. I deliberately drew their attention to the number patterns and emphasized the mathematics presented in the story.

What impact did my work in the classrooms have on the teachers? How did I meet them through this process of providing release time?

Teacher relations of *I-Thou*

At the beginning of the project, I offered the teachers an opportunity to remain in the classroom for my lesson. They did this to varying degrees. Martha used some of the time to contact parents and work on teaching tasks. Yet, she spent many lessons working at her desk and interspersing her comments with mine. She particularly enjoyed the stories and commented that the solutions that the students were creating were unexpected. Barb created a series of performance tasks for the mathematics unit she was teaching and worked with individuals and small groups of students. My presence in the classroom afforded the opportunity for her to develop some alternative assessment practices. Denise chose to work on administrative tasks, often related to her leadership role in the professional learning community adopted within her school setting. She appreciated the value I placed on her participation in our group and saw this as a reciprocating arrangement of time. Christine moved in and out of the classroom during my lessons. Because I taught her students at the end of the day, she often used the time to prepare the library for our group meetings.

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[12] See Appendix B for more examples of the stories I told to the students.

[13] According to Bruner (1986), the paradigmatic mode of thought emphasizes general causes, uses procedures to verify empirical truth, and is driven by logically generated hypotheses. Narrative ways of knowing focus on particular experiences, establish verisimilitude, and are driven by connections of lifelikeness. I discuss the distinction between narrative and paradigmatic aspects of mathematics further in Chapter 7.

I had anticipated that teachers would see my interpretation of the metaphor *mathematics is story.* Although they were not in the classroom for many of my lessons, they were provoked by the responses of their students. After working on the magic squares, Christine mentioned to me that her students were enthralled and attentive and that I "had them eating out of the palm of my hand." She commented that she had had no vision of how stories could be used in mathematics and saw for the first time how powerful it was. During various group meetings, all of the other teachers remarked about the impact of my teaching on their students. I was offering another possibility for *It* by working intensely on *I-Thou* relations with the teachers by providing this release time.

There were some unanticipated benefits to providing release time. This aspect of the project provided an opportunity for teachers to observe my implementation of historical stories and they could see the influence of this *It* on their students. I was very comfortable as a teacher in the beginning and my teaching background helped me to gain credibility with the teachers. There was a sense of home visitation: I was not just working in the classroom but encountering the teachers in their environment. I was able to form interpretations of the teachers' images of mathematics by seeing how they set up routines in the classroom, by observing how they taught, and by engaging in ongoing conversations of their weekly mathematics lessons. There was a reciprocal sharing of pedagogical knowing through these dialogical encounters.

I believe relations of *I-Thou* developed quickly because the teachers were placed in a situation that required their trust. Because of my intense involvement in their classrooms, they helped me negotiate the beginning relations with the students and frequently expressed their appreciation of my willingness to work with them at the schools. During the first few months, I was the common link between the teachers. I connected themes of our group conversations and extended these during my individual dialogues with each teacher. Much of the content of our group meetings was based on their classroom practice. I could relate to these because I was familiar with the students, the teachers, and the school context.

I felt like an expert when I was in the classroom. I saw myself as a "teaching" specialist and, as a former teacher, I was similar to these teachers. At the beginning of the study, I was able to enter with my whole being. Buber writes:

> The primary word *I-Thou* can be spoken only with the whole being. Concentration and fusion into the whole being can never take place through my agency, not can it ever take place without me. I become through my relation to the *Thou*; and as I become the *I,* I say *Thou.*
>
> All real living is meeting. (p. 26)

By providing release time, all learning was meeting for me.

Chapter 4 – Attempting Everest and other Metaphors

Mathematics is…

> *Martha: Mathematics is a molehill that can often become a mountain.*
>
> *Denise: I can relate to that.*
>
> *[laughter]*
>
> *Martha: However, proper training and equipment along with careful observation, planning, and thinking can lead the climber successfully to the top, over the top, and beyond. Liberating.*
>
> *Barb: That's neat.*
>
> *Gladys: I like the molehill and mountain.*
>
> *Barb: Yes. I do too.*
>
> *Martha: And that was me. When I was a kid, math was Mount Everest. It didn't work at all. Ever. In any way, shape, or form. And I managed to squeak my way through, and by memorizing stuff. I didn't understand it.*
>
> *(Second group meeting, September 9)*

I had initiated my research project in May by meeting individually with each teacher. Engaged in a conversation about teaching and learning mathematics, I had noticed that each teacher seemed to use specific phrases to describe mathematics. Many of these were metaphorical in nature. For example, Martha described trying to survive by beating mathematics. She realized that she could get lost and often needed to "wriggle her way out of it." She stressed the importance of learning skills and rules through repetitive practice and described aspects of her mathematics lessons as "drill and kill." She wanted to be involved in our project to learn new ways of "attacking" mathematics. The language Martha used to describe mathematics seemed consistent with a metaphor of *mathematics is a battle*. Martha spoke as if she viewed the learning of mathematics as winning or losing a battle, as attacking and beating an opponent, as requiring planned strategies for victory, and as defending her own position. If metaphors are more than simply ways of speaking, I could conjecture that she talked about mathematics in this way because she viewed it this way.

Barb referred to building and connecting when describing her students' learning of mathematics. Kim also emphasized building, connecting, and step-by-step methods of teaching. Denise spoke about teaching mathematics in terms of training and structuring as she delivered the lesson. These descriptions appeared to point to metaphor of *mathematics is a factory.* Assumptions about the nature of mathematics appeared throughout their language.

As we began the second group meeting[14], I introduced a reflective writing task. I wanted the teachers to consider how they thought about teaching and learning mathematics because I believed this would impact the stories they would create and the series of lessons they would design. After selecting descriptive words from the transcripts of the initial individual interviews, I presented a list of terms to the teachers (see Appendix D) and asked them reflect on these as they thought about how they viewed mathematics. I was interested in why they talked in metaphors and in how these conceptual ideas influenced their pedagogy. Lakoff and Johnson (1980) write:

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[14] At our first group meeting in June, we introduced ourselves to each other and shared stories about our teaching. We met in a restaurant and I did not record this conversation. The meeting offered an opportunity to begin relations of *I-Thou*, which we resumed in September.

> We have found [...] that metaphor is pervasive in everyday life, not just in language but in thought and action. Our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature. (p. 3)

I wanted to investigate their metaphors for mathematics in order to help me understand how they were viewing mathematics. The task I offered them was to complete the sentence, "Math is a ..." Explicit in the task was the notion of developing a metaphor of mathematics.

I encouraged them to express their views of mathematics to one another. The first two teachers who spoke did not explicitly employ metaphors. Denise suggested that teaching mathematics involved "modeling and structuring math in a guided, manipulating style classroom by instructing and facilitating learning to allow students to apply and build on their learning. Students will feel empowered." She chose not to elaborate and I indicated to Christine that it was her turn to talk.

Christine had not written a reflection. She admitted, "I had a lot of trouble with math when I was a kid. And I'm not good at teaching it. It is not my strength." She paused for a moment and then continued, "So this for me is quite a jump, to even do this. Even to talk about it is quite a jump." However reticent she appeared, she seemed anxious to contribute to the conversation as she offered her story of disconnection. She commented that sometimes she had no idea what she was teaching in mathematics and found that "quite scary." She said, "It's only grade 4 but I still have that difficulty. All this time. I don't relate to it at all. For some reason it doesn't have much connection for me."

The rest of her monologue included descriptions of mathematics as modeling, building, and framing. According to Christine, knowing mathematics was linked to relating to mathematical words and coming to know mathematics depended on relationships with a mentor who mediated the content: "I found that the relationships where somebody was building a personal relationship with me helped me to get it." Her conversation was punctuated with bursts of laughter that seemed to indicate she was nervous as she attempted to speak honestly with the group members. She found it difficult to talk about her views of mathematics and her vulnerability was evident as she described her previous experiences with mathematics. Yet, significantly for me, she acknowledged that relations of *I-Thou* helped her build stronger relationships with *It*.

Immediately following this revelation, other teachers began to report negative experiences of learning mathematics as children. Denise shared her angst when confronted with the symbols for less than and greater than. Common experiences of difficulty with mathematics arose. When Martha offered her metaphor *mathematics is Mount Everest*, a theme of separation was evident, as the mountain was definitely viewed as being outside of her.

After much conversation, Barb abandoned the metaphor of *mathematics is Mount Everest* by offering another perspective:

> Math is a bridge-building activity. You need to see where the bridge can start […] and then it goes onto a road that can lead you anywhere The materials you build it need to be strong, the bridge doesn't go up all at once, it takes time for it to go up, lots of people are involved.

While she was explaining her thoughts, the other teachers interrupted her. Denise pointed out that there were failures and successes. Barb suggested that "sometimes the machines don't work. And sometimes the workers show up late." Martha commented, "Sometimes the raw materials aren't available."

When Barb's metaphor of *mathematics is a bridge-building activity* was presented, this seemed to resonate with the other group members. She provided an image of modeling, building, and connecting and addressed the difficulties of understanding mathematics shared previously by all three teachers. The teachers began to consider the relationship between mathematics and a bridge. They encountered this metaphor as "a species of understanding, a form of seeing-as" (Zwicky, 2003, p. 4 left). A theme of connections emerged and persisted throughout the remainder of our conversations. This seemed to be a significant place of insight and a beginning site of relations of *I-Thou*.

Metaphoric Images

Metaphors[15] show relationships. Derived from the Latin word *metaphora* meaning 'to bear, to carry, to transfer', metaphors are linguistic expressions in which "a name or descriptive word or phrase is transferred to an object or action that is different from, but analogous to, that which is literally applicable" (*OED*, 1989).

Metaphors focus on similarities. Game and Metacalfe (1996) suggest, "Meaning is produced metaphorically, through a transfer between domains or terms. A transfer that in a way forces us to *make* sense" (p. 125, italics in original). When metaphors are presented, I respond by making sense of the similarities between the idea and the object. Gadamer (1989) comments:

> If a person transfers an expression from one thing to the other, [s/he] has in mind something that is common to both of them; but this in no way needs to be generic universality. Rather, [s/he] is following his widening experience, which looks for similarities, whether in the appearance of things or in their significance for us. The genius of verbal consciousness consists in being able to express these similarities. This is its fundamental metaphorical nature, and it is important to see that to regard the metaphorical use of a word as not its real sense is the prejudice of a theory of logic that is alien to language. (p. 429)

The analogous dimensions of metaphors prompt new ways of thinking about an idea. By noticing similarities between two things, a transfer of meaning can occur as new connections are formed. Pimm (1988) suggests that links between old ways and new ways of thinking are created by focusing attention on certain commonalities:

> The extended meaning may also have a certain retrospective effect on the old setting, by drawing attention to certain features of it that may not have been

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[15] While the literature focuses on metaphors, I have included the teachers' use of analogy and simile in my interpretation. To clarify this difference, I have used the phrase *mathematics is* to indicate a metaphor and have described *mathematics as* when writing about the teachers' descriptions of mathematics through the use of analogy and simile.

> noticed previously or considered of importance. It certainly has the effect of altering the balance between the various connotations of the concept. (p. 33)

Lakoff & Núñez (2000) write, "Metaphors do not just map preexisting elements of the source domain onto preexisting elements of the target domain. They can also *introduce new elements* into the target domain" (pp. 45-46, italics in original). Thus, metaphors have the power to shift our understandings of reality as new links between the idea and the object are considered.

In addition to creating new meanings, the transfer of a name or idea to an object or action forms a connection between abstract thought and concrete experience. Lakoff and Núñez suggest, "Metaphor is not a mere embellishment, it is the basic means by which abstract thought is made possible. One of the principal results in cognitive science is that abstract concepts are typically understood, via metaphor, in terms of more concrete concepts" (p. 39). Investigating metaphors of mathematics offers a means for understanding teachers' views of mathematics. Again, metaphors show relationships between *Thou* and *It*.

Metaphors link abstract ideas to concrete images, thus evoking an experiential connection. Metaphoric thought supports embodied knowing and is not merely a communication or visualization device. Game and Metcalfe (1996) comment that "when metaphor engages us, we respond through the emotions and memories that reverberate with the role. Our enactment of the world is a method acting informed and energised by the previous experiences that constitute us" (p. 51). Exploring metaphors of mathematics that arose within our conversations offers a means for understanding teachers' experiences of mathematics and how they might be connected to their images of mathematics.

Metaphoric thinking reflects Buber's notion of encounter. Metaphors encourage a shift from *I-It* to *I-Thou* as the *I* enters into a dialogue focused on similarities. Buber suggests that *I-Thou* relations recognize the possibilities of the space between us. This space, which he referred to as the sphere of the between, is negotiated through dialogic encounters of *I-Thou*. Differences of *I-It* are dissolved. Relations among *I*, *Thou*, and *It* do not accentuate differences; the uniqueness of *It* is not emphasized. Metaphors of *It* point to relations with *It*. Relations of *I-It* can shift to *I-Thou* through metaphors.

Problematizing the Metaphors

In my study, the teachers grappled with their views of mathematics, which they tended to express through metaphor. The dominant metaphors of our initial conversations were mathematics is a battle, a mountain, a factory, and a bridge. Each of these metaphors offers a particular view of mathematics. Why did these metaphors resonate with teachers' thinking about the nature of mathematics? What might these metaphors suggest about the teachers' relationships to *It*?

Mathematics is a Battle

The metaphor *mathematics is a battle* can be linked to experiences of fear, struggle, and even survival. Fear is a common theme among people who experience mathematics as a battle and is often viewed as something to be overcome (Tobias, 1994). In our second group meeting, Barb commented that many of her students lacked confidence and were overwhelmed when they attempted to solve word problems in mathematics. They did not know where to begin and felt like they were lost. The battle of mathematics was frustrating to her because the "curriculum is too full" and "there are too many concepts that they expect the children to learn at a certain age level." She suggested that sometimes their failure was "more just a panicky feeling that they get and they really do understand the problem but because they've had difficulty before, they look at it, too many words. They just shut down." Fear immobilized these students. She wanted her students to overcome their fear and to be confident in knowing they could do mathematics.

Childhood experiences with mathematics impacted some of the teachers. Denise did not like mathematics much because she was too frustrated and too overwhelmed. She commented that although she was more comfortable with mathematics now, she was not confident. She worried when her students were unable to understand the mathematics: "The minute you say *problem solving*, all the barriers go up and they don't want to talk about it or they don't want to do it." By providing fun and personal experiences for them, she attempted to help her students overcome their fear. Martha stated, "I did not like math as a kid. I proved I couldn't do it. I hated math in high school. I couldn't do it in high school. I proved myself right in university. I didn't get math at all."

The thread of struggle with mathematics was evident in the experiences of many of these teachers. Martha told us about her attempt to teach the Pythagorean Theorem to her students. She had spent an entire weekend looking at the textbook and drawing triangles. Although she had thought she knew what she was doing, when she started showing the students she got "turned around looking at the screen and the overhead and lost myself in the reverse view of things. I also lost my hypotenuse; I couldn't find it any more." She attempted to work through several more examples but this was not successful. She went home in tears, thinking she had destroyed her students.

To Martha, this experience seemed to capture her relationship to mathematics. She described her struggle to "relearn" the Pythagorean Theorem. As she independently retraced her steps in the textbook, she eventually found the elusive hypotenuse. She believed that people who got the mathematical answer had to "draw the pictures and get the calculator out and stutter and stammer their way through it." By using the terms *stutter* and *stammer*, her struggle with mathematics may have included aspects of fear or hesitancy. Certainly, her description indicated a lack of fluency with mathematics.

Christine struggled with providing an explanation for a mathematical concept to her students. She said, "Sometimes I can come up with it and the longer I do this the more I can get a better way to explain it, but I'm not very good at explaining the whys." She mentioned that she "never really got it either when I was younger so I always struggled with it." Despite her own struggles with mathematics, Denise did not know "why it has to be such a drudgery."

Mathematics seemed to be an enemy, something that took a great deal of work to understand. Martha used the terms *conquer* and *beat* to describe her successful experiences. This appeared to indicate an adversarial relationship with *It*. Christine also seemed to have a negative relationship with mathematics. After she noted the extra courses she enrolled in, the tutoring she had endured, and the teacher assistance she was given, she exclaimed, "For that amount of struggle […] it really bugged me that I couldn't get it." Although she tried to get it, "it took me until I didn't feel negative about it that I was actually able to do it."

Some teachers in the study felt they had survived their childhood experiences with mathematics. Christine revealed that "I never, ever enjoyed it" and that "I am only kind of getting to enjoy it the longer I teach it." Christine told us that her strategy for survival was to avoid mathematics. This was evident in her hesitancy to have me teach a mathematics lesson in her classroom. We had made the appropriate arrangements but when I arrived at the appointed time, she had sent her students outside to play in the snow. As I teased her about this being the ultimate way to avoid mathematics, she slowly smiled and agreed.

What I find interesting is her admission that she was starting to enjoy mathematics more as she taught it. Her willingness to participate in our project seemed to indicate that she was overcoming her fear and that she was winning the battle. After she told us that "even to talk about it is quite a jump," members of our group affirmed her risk-taking decision to share her personal story and acknowledged her vulnerability through this revelation. Denise commented that damaging attitudes toward mathematics were evident in part because of how children were treated in the past. Providing fun experiences were perceived by both Denise and Christine as helpful ways for overcoming fearful attitudes toward mathematics and being successful.

Mathematics is a Mountain

How might mathematics be like Mount Everest? In the opening conversation of this chapter, Martha offered a metaphor of the highest mountain on our planet to describe mathematics. Like Mount Everest, mathematics can be severely challenging. Many of the teachers in this study framed mathematics as difficult. Looming over them, mathematics was sometimes menacing. Christine commented that it was scary to think that she had no idea what she was doing when she taught it. Although she wished to "make some kind of relationship that [the students] connect with," she herself did not relate to mathematics. It remained an *It* and its height tended to loom ominously over her intimidated self. Denise stated that mathematics had made her cry as a child when she was confronted with less than and greater than symbols. This high-altitude world of mathematics can be cold. It could even be described as bitterly glacial and detached.

When mathematics is viewed as a mountain, its foundation is important in the climb from the base to the pinnacle. Christine commented that when the mathematical foundation was present, she was able to build on it. The idea that mathematics is promoted as "a base of knowledge" is evident in the language of the provincial mathematics curriculum. Also contained in the provincial document are mathematical concepts arranged in a progressively stepped curriculum, suggesting that mathematics is a predetermined mass that can be negotiated by climbing towards a visible goal. The metaphor, *mathematics is mountain*, can be interpreted as an immovable heap of isolated facts and arithmetic procedures with a solid foundation.

In the opening conversation of this chapter, Martha stated, "Proper training and equipment along with careful observation, planning, and thinking, can lead the climber successfully to the top, over the top, and beyond." When mathematics is viewed as a mountain, what constitutes careful planning, proper training, and suitable equipment?

Careful planning is necessary for success. Mountains are harsh and inhospitable. Mistakes can be deadly on the mountain and not having the correct answer can result in failure. From this perspective, knowing mathematics is dependent on getting the correct answer.

To *get* means to obtain or gain possession of something. What does it mean to possess the mountain of mathematics? In describing her relationship with the mountain Pedernal, Georgia O'Keefe states, "It belongs to me; God told me if I painted it enough, I could have it." What kind of relationship to the mountain does a painter have? The mountain is necessarily viewed as an object, as an *It*. It is distant from the artist. According to O'Keefe, it is acquired through repeated practice. Ownership is possible. It cannot be shared.

Often mathematics is experienced as a possession. Many times I have observed children closely guarding their mathematical possession as they shield their work from prying eyes by covering it with their arms and turning their backs to their classmates. The friends that possess the mathematical answers are not willing to share. They do not want their mathematics to be stolen. If a classmate copies their work, they give up possession of a precious secret. They give up power. To regain power, they are compelled to repeat the project of getting *It*.

For me, the possession of mathematics is linked to power and the ownership of knowledge. Mathematics is possessed and mastered through the use of reason and argument. Walkerdine (1988) questions the power and mastery of reason. She challenges the rational dream of perfect control in a perfectly correct and rational universe:

> My central point, therefore, is that this so-called natural process of mastery entails considerable and complex suppression. That suppression is both painful and extremely powerful. That power is pleasurable. It is the power of the triumph of reason over emotion, the fictional power over the practices of everyday life . . . The 'reasonable person,' in Piaget's terms, is 'in love with ideas' not bodies. (p. 186)

The experience of mastering mathematics may involve relations of separation and distance, a suppression of embodied encounters with *It*. Sometimes this experience of possession can be isolating.

Getting the correct answer depends on proper training. The mind must be conditioned and rules must be learned. There must be consistent practice through drills and repetition. Basic skills must be mastered. The climber must be able to persevere.

An emphasis on training was apparent in our conversations. Martha stated, "Addition is addition. Always. It doesn't matter. Addition of fractions: We have to have the same on the bottom. If we've got the same on the bottom [...] we just add the top." Denise recounted the steps of performing long division and commented, "And why did you do it? Just because. That was the way it was. That's how it's done." For these teachers, coming to know mathematics involved the simultaneous promise and denial of a reason for performing algorithms. In other words, the phrase *just because* seemed to start to point to a reason but instead became the end of a sentence. This could be considered an example of what Hewitt (1999) refers to when necessary concepts are treated as arbitrary. I explicate this in more detail in Chapter 5.

For Christine, proper training included using a step-by-step method of instruction and building from concrete to abstract concepts. When teaching a skill, she thought it was important to "keep going back to those things and repeating those things over the year." Martha's primary goal was "to get my kids to be able to add, subtract, multiply, and divide." Within this metaphor, coming to know mathematics entailed practicing skills in isolation and memorizing procedures. The climb involved progressively abstract steps on a predetermined path. Training was paramount.

The necessary equipment was provided by the teacher. Barb described this as a bag full of strategies: "Is this strategy called 'guess and test'? Is it called 'make a table'? What is the strategy called?" In addition, the suitability of the equipment was determined by the teacher. Barb commented, "You have to keep your tasks quite short and they have to be very specific, not very specific but clear. They have to know exactly what it is they're trying to accomplish." Again, the equipment used while coming to know mathematics reflected traditional views of mathematics and the responsibility for the expedition rested with the teacher.

Mathematics seen as a mountain reflects traditional notions of the discipline and is often experienced as *I-It* rather than *I-Thou*. In other words, mathematics is to be experienced, not encountered.

Mathematics is a Factory

While notions of step-by-step training and building were evident in the previous metaphors, the metaphor *mathematics is factory* offered a new dimension of mathematical usefulness. Utilitarian purposes for mathematics were important to the teachers. Barb, who had remained relatively quiet during the conversations of struggle and survival, contributed much to the discussion on the importance of mathematics. She commented, "I think it's very important that the children see the reference to real life." Whenever she taught mathematics, she tried to ask students where they would use it and why it was important to learn it. She thought this was crucial: "It's essential and it's used everyday but I think what children don't realize is how much they do use it everyday [...] It's very important for them to understand the connection between real life and what they're learning." Denise echoed, "Math is everywhere. One thing I know for sure. Everything you do or say can be turned into a math problem."

Reference to the utility of mathematics in "real life" is problematic for me. This term can imply that mathematics is estranged from reality. Although Barb seemed to emphasize the embeddedness of mathematics in everyday life, she interchanged the terms *real life* and *everyday life*. This might indicate that she grappled with the remoteness of mathematics and had to work at recognizing its function in everyday contexts.

The belief that mathematics is experienced in everyday life was a prevalent thread of the conversations. Lakoff and Núñez (2000) suggest, "One of the great findings of cognitive science is that our ideas are shaped by our bodily experiences—not in any simpleminded one-to-one way but indirectly, through the grounding of our entire conceptual systems in everyday life" (p. xiv). This seemed to be understood by these teachers, yet the language they used to describe their relationship with mathematics remained disembodied. Denise proposed that mathematics was necessary in order to "function in life. Because math is everywhere. Math is needed for everything that you do." Although statements such as these are often offered as platitudes, they emphasize that the functional view of mathematics is well established and commonly understood.

When mathematics is viewed solely in functional terms, it becomes a consumer product. The teachers in my study taught from curriculum documents utilizing language that could be interpreted to support the view of mathematics as a consumable. Here mathematics may be viewed as a commodity where students are expected to experience the "power and usefulness of mathematics" (Alberta Learning, 1997, p. 2). The content is stated as outcomes that are "measurable and identify what students are expected to know and do" (p. 13). Concepts are gradually developed linearly "from the concrete to the pictorial to the symbolic" (p. 13) and mathematical meaning is best developed "when learners encounter mathematical experiences that proceed from the simple to the complex" (p. 1). The assembly line of the factory runs continually within a functional perspective.

There was evidence that most of the teachers in my study interpreted the curriculum outcomes through functional lenses. One example emerged within our conversations about processes of subtraction. Denise recounted how she allowed children to use manipulatives in order to get the correct answer when subtracting. She emphasized the linear development of mathematical concepts as students moved from the concrete to the symbolic. Christine spoke about having her students explain the steps as they were performed. Although she believed she was encouraging her students to communicate their understanding of subtraction, her account of what they were doing involved regurgitation of the procedural steps for subtraction. Likewise, Martha's interpretation of the term *demonstrate* focused on an accurate proficiency in performing the subtraction algorithm. While I believe the writers of the curriculum documents intended for students to construct their own understanding of subtraction while engaged in a problem-solving context, many of the teachers believed that students should learn how to perform the algorithm without error. Their interpretation of problem solving was limited to writing complete sentence answers to story problems. I expand on this notion in Chapter 6.

In this particular curriculum, schooling is functional as students are prepared for productive lives in a "rapidly advancing, technological society" (p. 2). An embodied appreciation for mathematics is not expected, as students should only "show *some* enjoyment" (p. 2, italics mine) of mathematics by the end of the program.

Like factory workers, some of the teachers were focused on producing manufactured goods. The language they used indicated how they thought this could be achieved with their students. Denise wondered if she was "on the right track" and wanted to be sure she "delivered something and felt that they got it." The terms *right track* and *delivery* indicate that perhaps the assembly line has an optimal path for maximum efficiency and that teachers can convey mathematical knowledge to the students. Martha spoke of rewarding students with candy, implying that payment was the primary motivator for learning mathematics.

Relations of *I-It* are evident within this perspective. Mathematics is experienced as monotonous and routine. It is a means to an end as students learn the necessary applications to everyday life. In other words, students do mathematics to live rather than live to do mathematics.

Mathematics is a Bridge

When the metaphor *mathematics is a bridge-building activity* was offered near the end of our second meeting, it was taken up by the other teachers and extended. Their interactions involved attempts to confirm Barb's meaning of bridge and to add new elements. All teachers entered into this conversation as they investigated the links between mathematics and bridges. Communication through analogical thought seemed to prompt some teachers to look beyond their original view of mathematics to another view of mathematics as a connection. This particular conversation sparked much interaction and the notion of ***mathematics is a bridge*** became implicit at different times throughout our project as new relationships to *It* were considered.

For me, however, there is an inherent confusion in the metaphor. Is mathematics a bridge? If so, how is mathematics a bridge? Is mathematics seen as building bridges? If so, what does the bridge join together? And what is the bridge? Mathematics was described as a bridge-building *activity*. This suggests that mathematics is not an object, a notion quite different from those offered within previous metaphors.

Barb suggested that a bridge was constructed with strong materials, it was constructed over a long time period, and it was constructed by people. Perhaps mathematics can be seen as a construction project, an activity of building with robust ideas through a spiral curriculum of 12 years by teachers and students. Denise added the notion of failures and successes as being part of this activity. Obstacles to constructing the bridge included machines that did not work, workers who were late, and raw materials that were not available. I could interpret this to mean that sometimes the building tools or teaching strategies are not appropriate. Perhaps the workers or students are late because they are not developmentally ready for the mathematical concepts. Maybe the raw materials are not available because teachers lack mathematical knowledge and experience. These ideas are reminiscent of a view of mathematics as a commodity, yet there is evidence of human agency in the activity of the bridge building.

It is important to note that Barb did not say that mathematics was a bridge. Thus, she seemed to privilege the bridge-building activity. However, she did describe the bridge as leading somewhere and providing access to roads across the gap. In other words, the bridge provided a connection. The idea of connection was a consistent thread throughout the conversations: "The more connections that we can make for kids in making things fit together, I think the better they're going to do all around." If mathematics was viewed as a bridge-building activity, then perhaps mathematics was the activity of constructing connections. Since Barb emphasized the importance of connecting mathematics to the "real world", maybe the bridge was viewed as the connections between the learner and the world. In other words, mathematics was a connecting activity.

The notion that mathematics was the activity of making connections may have resonated with the teachers because of previous conversations identifying a need for connections. This image seemed to help some teachers express the separation from mathematics they had experienced. Perhaps the bridge-building metaphor provided a way of talking about the desire and need for robust *I-Thou* relations with mathematics.

Barb seemed to view mathematics in a way that was different from the perspectives of the other teachers in the study. She appeared to have experienced an *I-Thou* relation with mathematics. However, this was not evident in the metaphors offered by Martha, Christine, or Denise. *Mathematics is a battle*, *a mountain*, and *a factory* conjured up images consistent with traditional views of mathematics. While most of the teachers in this study would probably relate to the view of mathematics as an *It*, Barb offered an alternative image of mathematics. By encountering *mathematics is a bridge-building activity*, many of them reconsidered their previous experiences. Barb introduced human agency into mathematics. A strong desire to connect to mathematics was expressed by Christine, Martha, and Denise. Barb's metaphor seemed to convey this need, thereby creating resonance with them.

Near the end of the second group meeting, Barb wondered about the role of language in making connections to mathematics. The metaphor *mathematics is language* permeated our conversations throughout the next three group meetings. I explicate this metaphor in the next chapter.

Chapter 5 – Conversing Beyond Words

Mathematics is Language

Barb: Do we need the language first or the concept? Children often have the language but don't have the one-to-one correspondence [...] How did they keep place in a place value if they didn't invent zero and how did they, after they went after ten? But I guess you'd only have to worry about that when it was written. Because you wouldn't have to worry about it until it was written really. The only time, until it was represented [...] Because that's only when you really do need a zero, to hold the place value. Other than that, as I was saying here, there's no need for a zero. Because none, nil, is the concept. So we're back to written language. And written language and math. Because math has its own language.

Denise: But we don't always use it. I know when we were talking about [when] you go to the store about bananas. [The grocer] doesn't say, "We have zero bananas." He says, "We don't have any bananas" or "We have no bananas today." So we're not even using the word zero. And the kids, when you ask them a question, how many do you have left?, [they say,] "None." And I'll say, "How much is none?" "Zero." You have to encourage them to use the word.

Martha: I hadn't thought of that.

Christine: I guess I'm wondering how important it really is.

[laughter]

Denise: Tremendous.

Gladys: And I guess we have to ask the question, what is the zero for if it's not necessary to communicate?

Denise: Why do we use it? [...]

Barb: Well, I can see that it's really important in place value and it's certainly important when we're doing algorithms for adding and subtracting. They have to know that.

Denise: They have to know that word zero. And what it means. Instead of no or none.

Gladys: And so math is a language, it's a translation?

Barb: Oh, definitely.

(Third group meeting, September 23)

Mathematics is often viewed as a mountain of disconnected facts and isolated procedures and can be thought of as an abstract and highly formalized subject. Expressing mathematical ideas solely in symbolic form is often perceived as the best mode of mathematical communication. In the conversation above, the teachers were grappling with the relationship between language and mathematics. They recognized that the mathematics had its own language. There was a implicit distinction made between oral and written language, between speech and formalized mathematics, and between recognizing the symbol for zero and knowing its meaning.

Connections between mathematics and language have permeated philosophical discussions throughout history. Suggesting that the language of the universe is mathematical, Galileo (Galilei, Grassi, Guiducci, & Kepler, 1623/1960) writes:

> [The universe] cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it. (pp. 183-184)

Lacking the language is like "wandering about in a dark labyrinth" (p. 184). For me, blindly floundering and failing to negotiate the maze is similar to the view of mathematics as Mount Everest (see p. 63). According to Galileo, knowing the language of mathematics provides light for understanding and interacting with the universe. Language enables clear communication of the words of the world.

Using language to communicate provides a bridge to mathematical understanding. Pimm (1987) writes, "The general notion of communicative competence involves knowing how to use language to communicate in various social situations—how to use language appropriate to context" (p. 4). Indeed, communication among people is believed to be essential to mathematical understanding. Thus, teaching and learning mathematics seems to be intertwined with language. Durkin and Shire (1991) claim, "Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language" (p. 3).

While grappling with ideas of the origin of numbers, it seemed the teachers were making strong connections between language, communication, and mathematics. What is the relationship between communication and mathematics? How is mathematics a language?

Mathematical Discourses

At the beginning of this third meeting, I presented the first chapter from *Zero: Biography of a dangerous idea* (Seife, 2000) to the teachers. I chose to offer this selection because it contained extensive background information about the origin of zero and of various number systems and it was written in a narrative style that I felt was accessible to nonmathematical readers. As the teachers read individually, they noted any questions or comments that arose. When they had finished, I initiated a conversation about our responses. Our talk, presented at the beginning of this chapter, focused on the necessity of zero. Barb believed that the only reason zero was needed was to hold the place value when numbers were written. There was no other need for a zero because "none, nil is the concept." She suggested that mathematics had its own language. Denise acknowledged that students needed to be encouraged to use the word *zero*. How does mathematics have its own language? In what ways are mathematics and language similar?

Language can be described as a systematic method of communicating through oral and written words. It consists primarily of vocabulary and grammar. The vocabulary of a language is the words; the grammar of a language addresses sentence structure and rules for combining words. If the purpose of mathematics is to communicate, then what is the relationship between oral and written words? How can the grammar and vocabulary of mathematics be described? What is the language of mathematics?

Relations of Oral and Written Language

Pimm (1991) suggests, "Mathematics, when spoken, emerges in a natural language: when written, it makes varied use of a complex, rule-governed writing system mainly separate from that of the natural language into which it can be read" (p. 20). Sometimes mathematical language is viewed as consisting solely of numerals. When spoken, mathematical language also contains words that express quantities, such as the term *none*. Denise distinguished between number concepts of spoken language and written numbers. Martha reiterated this idea: "Zero didn't come to be until there was a need. We don't use *zero*, the word *zero*, in our language and conversations about math. Often we refer to it as *none*. We use a different word." Claiming that the only time we needed a zero was when we established a base-ten place value system, Barb commented, "Someone decided that there was going to be numbers to represent place, put a place value base-ten. Until that point, we really didn't need zero. They had one more and one more and one more." Some of these teachers appear to be grappling with the relationship between mathematical concepts of quantity emerging in natural language and the language used in formal mathematical systems. They would probably agree with Alfred North Whitehead:

> The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought. (cited in Seife, 2000, p. 6)

What does it mean to know the words? Differences between written and oral language in mathematics are evident in understanding what zero is. Barb and Denise seemed to be talking about *zero* in two different ways. Barb stated that zero was a placeholder. In this context, zero was perceived as a digit.

This notion of zero first emerged in the ancient Babylonian system of numbers. Unlike the Egyptians who used pictures and the Greeks who used letters, the Babylonians used two marks: a wedge to denote 1 and a double wedge to denote 10. Because they relied on the position of the marks to represent a number, they invented a placeholder using the mark of two slanted wedges. Thus, "zero was born out the need to give any given sequence of Babylonian digits a unique, permanent meaning" (Seife, p. 15). Barb linked the digit of zero to the written language of mathematics. Zero as a digit means that it has no value; it is not a number.

Unlike Barb, Denise referred to zero as a number, not a digit. In this context, zero has a value. It is a concept of nothing. It can be compared to other numbers. Anthony and Walshaw (2004) present their reasons for why the concept of zero is problematic:

> Although children may understand the idea that one has no apples, the idea that *no apples*, *no sheep*, and *no people* are the same because they have the same cardinality is very abstract—even through it naturally follows the idea that two collections of different objects can have the same cardinality. (p. 38, italics in original)

Because the term *zero* is not used frequently in oral language, it is often not part of early number experiences. Thus, zero may not be perceived as a legitimate number. Coupled with the idea of zero as a placeholder emerging from the written language of mathematics, the understanding of zero as a number can be limited.

Christine wondered about the importance of using the term *zero*. If we used the term *zero* in order to communicate, why could we not rely on the terms *none* or *no*? Barb attempted to explain, "Well, I can see that it's really important in place value and it's certainly important when we're doing algorithms for adding and subtracting. They have to know that." Denise insisted that it was tremendously important for the students to know the word *zero* and what it meant. Why is this important?

Zero as a placeholder is arbitrary, as is the name of zero. According to Hewitt (1999), names, labels, and symbols are true because of conventions. These symbols must be memorized and cannot be derived from the context. Originally called a cipher, zero is a symbol that has no value. Although zero may have no value of its own, it influences the value of other numerals and can increase or decrease these values according to its position. Yet, it is arbitrary. Hewitt says:

> I describe something as *arbitrary* if someone could only come to know it to be true by being informed of it by some external means—whether by a teacher, a book, the internet, etc. If something is arbitrary, then it is arbitrary for all learners, and needs to be memorised to be known. (p. 3, italics in original)

When Denise insisted that she had to "encourage them to use the word," she was bumping up against the arbitrary nature of zero. She needed to inform them of the label and the place-value system.

While Denise stated that her students had to know the word zero, she also insisted that they also had to know what it meant. For her, it seemed inadequate to view zero merely as a placeholder. Zero represented void and empty space and she was indicating that "nothing" could be signified. Here the *concept* of zero is not arbitrary but necessary. Hewitt distinguishes between arbitrary and necessary aspects of mathematics:

> There are aspects of the mathematics curriculum where students do not need to be informed. These are things which students can work out for themselves and know to be correct. They are parts of the mathematics curriculum which are not social conventions but rather are properties which can be worked out from what someone already knows. (p. 4)

Our confusion over the oral and written language of *zero* seems to indicate a need to consider both the arbitrary and necessary dimensions of this term. Rotman (1987) writes:

> Zero signifying an absence of signs and 'nothing' signifying an absence of things seem to occupy the same space once one denies the illusion that 'things' are anterior to signs. But zero and 'nothing' are manifestly separate, moving in different historical and linguistic terrains, and being carried by very different signifiers: against words—'nothing', 'rien', 'nihil', and so on—zero, represented by the symbol '0', has a physical shape, a graphic presence independent of particular languages, with its own iconography. (p. 58)

In the opening conversation of this chapter, Barb wondered about the relationship between the language and the concept. Does the language come before or after the concept? How are words and concepts related?

As we grappled with tensions of the arbitrary nature of vocabulary and the necessary nature of conceptual understanding, we delved deeper into the consideration of mathematics as language. Our conversations throughout the next three meetings focused on mathematical vocabulary. In the sixth group meeting, we spent time talking about what the terms *few* and *a couple* mean. Denise suggested, "We were saying everybody has a different view of what few means or what a couple is. You say a couple of cookies and they take three. That's not a couple." There was general agreement as we nodded. But we were still perplexed by the various understandings of these quantities. How did we know what a couple was? How much was a couple? Martha commented, "A couple is two because a couple is somebody who's married." However, Denise was not convinced as she explained how she understood the idea of couple: "For me it was just a word in the house: 'Okay, you can have a couple of cookies.' You just knew that was two. If I took too many, that was not a couple. It was just a word." How is a couple just a word?

When zero is used as a placeholder, the language of mathematics involves a system of arbitrary symbols, a grammar and a vocabulary. When zero represents a number, the language of mathematics involves a set of meanings. Both aspects are addressed by Halliday's notion of the mathematics register.

A Register of Mathematics

Halliday (1978) writes,

> A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195).

He claims that every language includes some mathematical meanings that offer a starting place for the initial learning of mathematics. However, the overlap between mathematical language and what I might call everyday English can contribute to misunderstandings. Denise was perplexed when her students did not understand the word *set*. When I suggested that they probably had not come across the term in their lives outside of school, she quickly pointed out that they had heard about different sets such as volleyball sets and setting the table. Barb proposed that they might have heard about a stage set. Confusion can occur when words share the same form but have distinct meanings. If students are not informed of the specific mathematical meanings, they will most likely attempt to connect the term with a meaning they are already familiar with.

Miscommunication can also occur when mathematical words and English words are pronounced the same but have distinct meanings. This occurred in our conversations when *some* was misunderstood as *sum*. Christine needed clarification of what had been said and asked, "Why isn't it S-U-M? Why is it S-O-M-E?" In the context of our conversation, this substitution was legitimate. We resorted to spell the terms when we thought there might be confusion.

One difficulty, then, in learning the mathematics register is the differences between the language of ordinary English and mathematics. Durkin and Shire (1991) write:

> Instances of such variation include words which have an altered meaning or grammatical function. In everyday usage, they function like adjectives: one book, two crocodiles, nineteen tower blocks. In mathematical discourse, they become nouns, constituting one of the first sets of mathematical objects, which in turn have properties like being odd or even, or prime. (p. 19)

The registers of mathematics and ordinary English are not mutually exclusive. The language of mathematics is not "clearcut or precise. It is a natural human creation, and…is inherently messy" (Halliday, p. 203). Existing words are borrowed and reinterpreted. New words are created. The registers overlap. Thus, a challenge of viewing mathematics as language is related to communication. Durkin and Shire comment, "Spoken language is a growing and evolving phenomenon, but the language of mathematics is written, symbolic and virtually static. This leads inevitably to changing or multiple relationships between the spoken words and the written symbols" (p. 158).

Throughout the study, specific questions about the grammar of mathematics did not emerge. However, during the third group meeting, most of the teachers were preoccupied with how to teach mathematical vocabulary. Denise struggled with the term *sum*. Martha recounted how when she announced that her students were going to subtract, she felt as if she had been "hit in the face" by twenty-eight blank faces. They had not linked the term *subtraction* to the phrase *take away*. She described her solution: "So I say, 'Yes, it's a subtraction question.' So they're just hearing the other word. I'm not saying, "No, it's not take away" or "Yeah, it is but we call that something else." Barb told us that she interchanged the terms *sum*, *total*, and *all together* when talking with her students.

These teachers recognized the importance of informing the students but none made an explicit attempt to help the students memorize this arbitrary vocabulary. They seemed to expect the students to make explicit connections between terms as they were implicitly interchanged. Although this might appear to offer a context for learning the vocabulary, Hewitt would suggest that this process be made more deliberately obvious for the students by assisting memory. If the teacher does not inform students of the arbitrary, students have to invent the meanings. This can lead to misunderstandings and can interfere with relations to *It*.

While I note that the teachers were preoccupied with vocabulary, there is a danger in emphasizing this aspect of the register. By privileging the arbitrary nature of vocabulary, the necessary aspects of mathematics can be forgotten. The mathematics register consists not only of vocabulary and grammar but of the meanings they express. Pimm (1987) writes:

> It is not just the use of technical terms, which can sound like jargon to the non-speaker, but also certain phrases and even characteristic modes of arguing that constitute a register. Part of learning mathematics is learning to speak like a mathematician, that is, acquiring control over the mathematics register. (p. 76)

Learning the mathematics register involves attending to both the arbitrary and necessary aspects of mathematics. It entails negotiating the oral and written modes of communication. And it includes distinguishing between the registers of mathematics and English.

When Barb asked, "Do we need the language first or the concept?", I was perplexed. To me it seems that the two are completely intertwined. Language is conceptual. Mathematics as language implies a communication of thought. To explicate the relationship between the language and the concept, I consider the discourse of mathematics, a communication of thought through action.

A Discourse of Mathematics

Mathematics as language implies that *It* is a human creation. Unlike platonic views of mathematics as a mountain, mathematics is seen as dynamic and enacted. It is relational as we communicate to "bring forth a world" (Maturana & Varela, 1987, p. 26). Gordon Calvert (2001) writes:

> Discourse, then, is not used simply to communicate nor is it used to construct a *view* of reality as though a 'real' world exists and is visible outside of us…Through a mathematical discourse we bring into being mathematical objects, concepts, theories, and proofs to make sense of our experiences" (p. 3).

Barb appeared to grapple with the dichotomy between vocabulary and conceptual understanding. However, when she described the importance of having her students talk in complete mathematical sentences, she said, "You have to tell me in complete mathematical language, what it was that you were doing…If you think of it in correlation to language, that's like having an incomplete thought." One interpretation of Barb's response might be that language and mathematics communicate thought. Discourse is often defined as "communication of thought by speech" or as a "mutual intercourse of language" (OED, 1989). Here, mathematics is viewed as a dialogic endeavour.

Sfard (2000) writes, "Knowledge viewed as an aspect of a discursive activity is no longer a disembodied, impersonal set of propositions, the exact nature of which is a matter of 'the true shape' of the real world; rather it is now a human construction" (p. 161). The relational context of social interaction through communication is emphasized when describing mathematics as a discourse. This seems reminiscent of Buber's notion of dialogic encounter. To me, this seems different from learning the mathematics register. I can experience the language of mathematics through this register or I can encounter the discourse of mathematics through relations of *I-Thou-It*. How is mathematics as language experienced? How is *It* encountered?

Learning the Language of Mathematics

The metaphor *mathematics is language* challenges the notion that action and experience precede thought. Conceptual understanding and language are linked and cannot be separated from experience. Pimm (1986) states, "With mathematics it is important to perceive language as experience as well as mediator between experience and the individual" (p. 51). Mathematics can be experienced as foreign, universal, and visual languages.

Our conversations throughout the first two months were punctuated with stories of mathematical misunderstanding. In the fourth meeting, the problem of understanding word problems arose. Martha recalled her experience of writing the grade 10 final exam and being unable to answer a typical train problem: Train A left California at a certain time. Train B left New York at a particular time. Where did they meet? Martha wrote, "On the track there were fifteen casualties." She did not understand the question. Many other teachers expressed their confusion when faced with word problems. Often these needed to be translated or explained to them.

Although mathematics appears to be written in English, in many instances it is experienced as a foreign language. It is not easily understood and often requires an interpreter. The difficulties of learning a foreign language are similar to learning the mathematics register. For many people, the movement between this register and the English register presents difficulties. These difficulties are not limited to vocabulary or grammar but extend to challenges with characteristic forms of problem solving. Understanding word problems is such an example.

During the fifth group meeting, Christine asked, "When did the vocabulary for math come along?" This question provoked much interaction within the group. Denise thought that "it came with speech, once they stopped grunting." Our conversation oscillated between problems when using language to describe quantities and how numbers had specific meanings. Martha told a story of how she argued with her sister about the difference between *a couple* and *a few*.

This conversation is just one example of an experience where words mean different things to different people. The vocabulary is linked with the meaning of the term and this can contribute to misunderstanding if the meanings are not socially negotiated. Experiencing mathematics as a foreign language can be frustrating and can lead to a lack of confidence when interacting in a mathematical context.

During the third group meeting, we were talking about ancient civilizations' creation of calendars. The notion that calendars were not read the same way by all civilizations was raised. Barb referred to her experiences of teaching in Australia where the concepts were similar because they were using a base-10 system of numbers. Martha mentioned that when she was living overseas, she went to a German-speaking school. Although she did not understand German, she found she could understand the mathematics. Clearly, her experiences suggested to her that mathematics was a universal language.

I recounted a conversation I had had with one of their principals who told me that mathematics was universal and believed it was the same everywhere. He was upset with one of the teachers for presenting an alternative algorithm for subtracting numbers. He did not feel we could expect children to be able to do this a different way but wanted his staff to teach children the conventional algorithm because that was the way the world did it. There was much disagreement among the teachers in this study about this sense of universality. Brissenden (1988) writes:

> Precision is clearly an essential feature of mathematics, particularly in calculations and algebraic manipulations, or logical processes. But far too many non-mathematical teachers are ready to accept uncritically the idea that mathematics is some rigidly-defined, unchanging body of knowledge, and that in consequence there is a 'correct mathematical language' on which all mathematicians agree. A better view is that mathematics is a 'way of knowing'—an activity undertaken by people, including learners—where results and procedures are constantly developing. (p. 7)

When mathematics is viewed as a universal language, there may be a tendency to envision mathematics as static and unchanging. Rather than accepting all aspects of the mathematics register as universal, I find it helpful to consider the *ideas* of mathematics as universal. Barb suggested that numbers expressed ideas and that "math is also communicating the idea to others." She identified this as a universal system because the numbers and procedures of mathematics were invented to express ideas. For example, if addition is understood as accumulating more, then numbers are just the expression of this idea. Thus, it is the ideas that are universal, not the systems of communication.

During our sixth group meeting, we talked about how students saw the base-10 blocks used as manipulatives for demonstrating place value concepts. Denise commented, "They seem to get stuck on the rod part. They'll see the three, like in thirty, and they'll just write it down as three hundred and then three and two. They're still saying three instead of thirty." Clearly, the image of the rods, the language used to describe the image, and the concept of place value seemed connected to Denise. Mathematics appeared to be a visual language.

She questioned what she should be saying in order to help students visualize the number. Barb suggested that she reinforced place value concepts when demonstrating the addition algorithm: "I never say 'five plus one.' I always say, 'in the ones place we have five and one, in the tens place we have this, in the hundreds place we have this.' So they get a more visual [image]." Martha wanted to know if this was a " language thing" or a "math thing". Our conversation investigated some similarities of these registers as we talked about the structure of the words for numbers in other languages.

Miura et al. (1994) investigate such similarities between spoken and written numbers and suggest that numerical names in Asian languages are congruent with their numeration systems. For example, 11 is spoken as "ten one" and 47 as "four-ten seven" in Chinese-based languages. Unlike most European systems of irregular number words, Asian number-naming systems explicitly support children's understanding of place value, base-10 number systems, and arithmetical computations. The teachers in my study seemed to be grappling with the relationship between number systems and the language used to communicate mathematically.

Another tension that teachers experienced was linked to talking about borrowing and carrying when subtracting and adding. Denise suggested, "You're supposed to call it *regrouping*. And we're supposed to teach parents to teach them *regrouping* because that's what we're telling them in school and when you say *borrow* and *carry*, they aren't going to understand you. But I do it all. I use all those terms." Barb agreed and stated that she explicitly told her students that their parents would probably use the word *borrow*. Martha refused to change: "We only talk about borrow and carry here so it makes sense in my head."

The focus here seemed to be on communication. The terminology had changed but parents and students were unfamiliar with this new word. What I found interesting was the lack of concern with why the term had changed. When I questioned them about this, it was not taken up by any of the teachers. Pimm (1995) writes:

> Certain images deliberately offered or unwittingly invoked when talking about what we are doing in mathematics can be quite curious. When subtracting, the operation of 'carrying' is sometimes talked about in terms of milk bottles and 'paying back on the doorstep' or 'borrowing' from the 'next door' or

> 'neighbouring' column as one might a bowl of sugar. Do these classroom ways of speaking sometimes get the better of us? Why do we continue to use them? (p. 2)

Although the language of the curriculum had changed, the teachers did not seem to contemplate the images these changes could evoke.

The view of *It* as language is interwoven with both our experiences of mathematics and our coming to know mathematics. In the next section, I investigate these connections.

Communicating Mathematically

Mathematics is language focuses on communication. This communication can be disrupted by confusion with the mathematics register or misunderstandings within mathematical discourses. Viewing mathematics as a language can imply that translation and interpretation are necessary in order to communicate.

When I asked the teachers if mathematics were a translation, this appeared to resonate and suggested that many were thinking about mathematics as a foreign language or a written communication in a second language. Christine linked her inability to solve mathematical problems to getting "stuck on the words." Although she could memorize the steps, she did not know how to proceed if the words were unclear. When describing how she coped with this difficulty, she said, "I have to go to another person, and say to them, 'Put this in other words for me. Somehow unpack it for me so that I know to talk about it.'" Christine identified the necessity for someone to translate the words of mathematics into a language she could understand. According to her, she could not participate in the discourse of mathematics because she could not speak the language. She depended on a translator to "unpack" the language and her relationship with *It* depended on mediation through a *Thou*.

Usiskin (1996) claims that one reason students have difficulty learning the mathematics language is that "mathematics is a foreign language for many students, for it is learned almost entirely in school and it is not spoken at home" (p. 238). Immersion in the language of mathematics at school thus becomes necessary for learning to participate in the discourse. The language is not learned unless it becomes embodied.

Viewing mathematics as language influences how teachers prompt this encounter. Brissenden (1988) suggests:

> It is, of course, part of the teacher's role to inject mathematical terminology and usage into discussions with her or his pupils. This process has to be seen as one of *negotiation of meaning* between teacher and pupils rather than one of imitation by the pupils of what the teacher says. In the former, formal definitions evolve as outcomes, not as dogmatic starting points which pupils struggle to accept without being able to appreciate the underlying reasons for the choices which have been made. (p. 7)

This view of mathematics sharply differs from *mathematics is a mountain*. Knowing mathematics is through participation, not acquisition. Social construction of meaning is emphasized here.

Providing opportunities for communication becomes a priority when viewing mathematics as language. This depends on strong *I-Thou-It* relations. Barb said that when she was talking about addition, she continually asked her students what their idea was and what they were seeing. She focused on their interpretation of mathematics and provided the space for conversation about how they were coming to know mathematics as she listened to their explanations.

The thread of explanation was taken up by Christine. However, her emphasis seemed to be on helping students memorize the steps. She described how she had tried to have her students talk through division. Working in partners, she asked them to take two steps of long division and "talk it in their head as they were doing it." She acknowledged that the only way she figured out long division was to keep repeating the steps in her head.

This was quite different from Barb's notion of explanation. Denise attempted to distinguish between explaining the procedure for long division and explaining the ideas of mathematical thought. She commented that although she could explain the steps, she did not know what they meant. In Christine's story of communication through explanation, there was no dialogical interaction. Working in dyads, she wanted her students to "talk in their head." Although this may have communicated an ability to memorize the steps through repetition, this failed to demonstrate how her students were participating in the discourse of mathematics.

Brissenden suggests that teachers engage their students in meaningful discussions as a way of promoting communication within the mathematics classroom. He emphasizes the need for distributed authority, good listeners, active participation, and the opportunity for building and testing the ideas of others. These characteristics are not evident in Christine's description of her students' talk. Coming to know the mathematics involves learning the language and communicating with others using this language. Both aspects are addressed by Brissenden. By encouraging his students to present and support conjectures, he is initiating his students into the processes of mathematics. By providing opportunities for them to participate in the conversation, he is immersing them in the vocabulary and grammar of the language.

The tenor of discussions is tentative. Explanations are offered to communicate our understanding of mathematics at a particular moment in time. They are dynamic. Gordon Calvert (2001) writes:

> Our explanations are not of an independent world, but rather, explain our own experiences in coexistence with others. The explanations we accept give rise to new experiences, new questions, and new explanations in a recursive cycle. (p. 57)

An inherent danger in viewing mathematics as language is the privileging of the authorized conventions. Meaningful discussions and conversations can be constrained by a requirement to use the correct terminology when communicating. Martha shared an experience she had with her four-year-old son as he was reading a 1-800 number advertised on television. He had read the digit zero as the letter 'O.' She had corrected him and had repeated it. She believed that is was important to correct him because Denise had raised this issue in one of our conversations. However, she did not seem to understand why this was important. She linked her son's ability to distinguish between the digit zero and the letter 'O' to future success in mathematics: "Maybe if I can start him at age four understanding math, then maybe he'll be farther ahead in math." For Martha, the goal of speaking the correct language was to be further ahead, not to improve communication.

Although Martha seemed to emphasize the official terminology, she was aware of the connection between communication and mathematics. In our fifth meeting, Martha suggested, "Math has evolved out of the necessity to clearly communicate. To be understood is important." The thread of communication was taken up by the Christine who believed that "the clarity of how you express yourself is key" and that "if you're going to talk about any kind of quantity, you don't have any idea of what's going on if you don't have something clear."

The teachers' emphasis on language and communication within teaching and learning mathematics opened up possibilities for considering mathematics as a humanity. In other words, previous conversations within our group prompted new perspectives of the nature of mathematics, experiences with mathematics, and coming to know mathematics. The teachers had investigated *mathematics is a language*. Reconsidering their image of *mathematics is Mount Everest*, many of them were becoming more comfortable with alternative views of mathematics. I offered the metaphor *mathematics is story* as a way of considering mathematics as a narrative language. Thus, m*athematics is language* provided a bridge to *mathematics is story*. This new thread of our conversation is explicated in Chapter 6.

When I consider the metaphor *mathematics is language* I wonder about the possibilities for *I-Thou-It* relations. In many instances, *I-Thou* relations involve a translator or someone who is aware of the mathematical possibilities. To what extent can a translator participate in an *I-Thou* relationship when access to *It* by *Thou* is dependent on the expertise of this *I*? I offer my reflections on this notion of expertise in Interlude 2.

Interlude 2 – Working on relations of *I-It* through attention to *Thou*

Relating to *It*

> *I consider a tree.*
>
> *I can look on it as a picture: stiff column in a shock of light, or splash of green shot with the delicate blue and silver of the background.*
>
> *I can perceive it as movement: flowing veins on clinging, pressing pith, suck of the roots, breathing of the leaves, ceaseless commerce with earth and air—and the obscure growth itself.*
>
> *I can classify it in a species and study it as a type in its structure and mode of life.*
>
> *I can subdue its actual presence and form so sternly that I recognize it only as an expression of law—of the laws in accordance with which a constant opposition of forces is continually adjusted, or of those in accordance with which the component substances mingle and separate.*
>
> *I can dissipate it and perpetuate it in number, in pure numerical relation.*
>
> *In all this the tree remains my object, occupies space and time, and has its nature and constitution.*
>
> *It can, however, also come about, if I have both will and grace, that in considering the tree I become bound up in relation to it. The tree is no longer* It. *I have been seized by the power of exclusiveness.*
>
> *(Buber, 1923/1958, pp. 22-23)*

There have been periods in my life when I have both experienced and encountered mathematics in the ways Buber considers his relation to a tree. For me, mathematics was sometimes experienced as an *It.* I learned to classify it as a science, "recognize it only as an expression of law," and perpetuate it solely in number. This was very much how I related to *It* for most of my childhood.

Change permeated my early years of schooling as I lived in numerous houses and attended eight schools in my initial ten years of education. What provided stability in my life were family and school communities. I became at ease in schools; perhaps the structure and routine helped me cope with childhood transitions. Indeed, I believe my study of mathematics, in particular, provided a sense of stability. In my experiences with school mathematics, the rules were stated and consistent. Answers were predictable, ambiguity was lessened, and class formats were similar. In these classes, I did not need to find a partner or group to work with. This was a relief to me since I was often the new student in the class and did not know anyone. The mathematical knowledge I had learned in other schools transferred to these new situations. In mathematics, I was successful.

At the beginning of my schooling, mathematics was experienced as an *It*. As I became more at ease with the subject, I became "bound up in relation" to *It.* I began to recognize mathematical patterns and developed an aptitude for solving logic puzzles and word problems. Mathematics became a familiar friend, one who moved with me in the first ten years of my school career.

When my family's movement stabilized, I became part of an established community. However, my overt interest in mathematics became an alienating experience. In high school and university, I would not discuss my grades in mathematics and if directly asked, I would reduce them by 20% so I was not distanced from my peers. In many social contexts, I was embarrassed by my ability. When I decided to become a teacher, I chose to study mathematics because of pragmatic reasons (the job market looked better for mathematics teachers than English teachers). However, I was still concerned that I would be alienated because of this choice. To me, pursuing an education degree with a major in English seemed more socially acceptable and certainly would have provided more possibilities for me to develop female friendships. Choosing to pursue mathematics education placed me in large classes with primarily male students. This created distance, yet I was still intrigued and challenged by mathematical problems. I enjoyed playing with mathematical concepts and puzzles and grew to believe that I could bring this passion to the classroom. Even though I had a robust background in mathematics, my relationship with *It* was strongly linked with *I-Thou* relations. In many instances, *I-It* relations were negatively correlated to *I-Thou* relations.

One exception occurred at the beginning of my teaching career when *I-Thou-It* relations were mutually strong. My background in mathematics provided an entrance for me into the teaching profession in an era of teacher surplus. For the first time in my life, I returned to the same school year after year. I loved teaching and I enjoyed working with my colleagues. I formed lasting friendships and liked sharing my mathematical enthusiasm and expertise with my students. Mathematics became my gateway into the community of teachers.

Throughout the years, my relationship with mathematics oscillated between this sense of encounter with mathematics and my experiences of mathematics as an *It*. After beginning my PhD studies in mathematics education, I often felt like a fraud. I wondered when my supervisors would discover that I was not a mathematical genius. Sometimes I enjoyed the awe (and fear) I inspired in people when I told them I was studying mathematics. I seemed powerful. Then I really felt like a fraud.

I was bothered by negative comments made by adults about their experiences with mathematics. I disliked discussions pitting qualitative work against quantitative understandings. I believed mathematics, a language of numbers and patterns, could be used to improve our understandings of human relationships. I sensed most people did not view mathematics in this way. Indeed, many mathematicians did not view statistical analysis as pure mathematics. By suggesting that mathematics could help study human experiences, I was uncomfortable. How much did I understand about this process? What were the possibilities for mathematics to be seen as a human invention? What made me an expert? Again, I felt like a fraud.

Finding a community within this context influenced my self-identification as a mathematics educator. During my PhD program, I struggled with my relationship to *It*. I wanted to be validated by the mathematics community as a mathematics educator, yet I resisted perpetuating the view held by many people that mathematics was elite. My unease as an expert in mathematics education was mitigated by my comfort as a teacher.

Throughout this study, I was conflicted because the teachers viewed me as a mathematics expert and I resisted this role. So strong was my desire for *I-Thou* relations, that I was willing to temporarily relinquish my relationship to *It*. Yet because of my interactions with the teachers, I came to relate to mathematics in a deeper way. This community of learners challenged my complacent notions of the ease of co-construction and I encountered tensions and joys in relation to these teachers as I sought to become a full participant in this study. Thus, I was able to work intensely on relations of *I-It* while working on relations of *I-Thou*.

I did not want to exacerbate the distancing experiences with mathematics that many of the teachers had talked about in their initial interviews so I chose to participate as a teacher. I wanted to learn about the history of mathematics alongside them. In other words, in order to avoid being viewed as an expert, I purposely disengaged from my own study of the history of mathematics until our group meetings began. In retrospect, this may not have been a fruitful decision.

Relations of *It-It* and *Us-Them*

I believe my resistance to being viewed as an expert was rooted in my learning experiences during my undergraduate studies. Two layers of expertness were evident: the pure mathematician as an expert and the professor as an "ivory-tower" expert. I did not seek to be either of these types of expert.

Majoring in mathematics, I completed eleven mathematics courses (three more than the required number for a Bachelor of Science degree). I enjoyed learning but grew disenchanted with some of the professors of these courses. One particular example is noteworthy. I was one of three people enrolled in a senior-level mathematics course. The instructor walked in, wrote notes on the board for the entire hour, and handed us a copy of these notes before departing from the classroom. He did not allow time for questions or discussion. In fact, I do not believe he knew my name; he certainly never addressed me. Clearly he had a relationship to *It*, but he seemed distant and inaccessible to me. As a pure mathematician, he appeared to shun *I-Thou* relations.

In the Prologue to Buber's book, translator Walter Kaufmann identifies people whose lives are dominated by the object of their interest as living in the world of *It-It*. These people have "no time for themselves, with no time to have a self." He writes:

> For all that, their "subject" is no subject in its own right, like a person. It has no subjectivity. It does not speak to them. It is a subject one has chosen to study—one of the subjects that one may legitimately choose, and there may be others working on the same subject, possibly on a slightly different aspect of it, and one respects them insofar as they, too, have no selves and are objective. (pp. 12-13)

When I first read this passage, my experiences of learning from some mathematicians resonated strongly with Kaufmann's description of the world of *It-It*. Not only was the *I* missing from this world, there was no *Thou*. My view of some mathematical experts was strongly connected to the world of *It-It*. I did not desire to live in this world.

The notion of distance was also evident in impressions I had of professors who lived in an 'ivory tower.' This term, used often to describe academics, hints at a separation from the world and sets up relations of *Us-Them*. I remember encountering this term during my first student-teaching practicum. It was used by my teacher associate to describe reasons for the tension between the expectations of my professor and the realities of working in a classroom. When I was inviting participants to be part of this project, one teacher referred to me as being from the ivory tower. He was pointing to the distance between himself as a teacher and me as a researcher. While Buber does not explicitly address this type of relationship, I believe it is reflected in his notion of *I-It* where the *It* is experienced and used but not encountered in mutual dialogue. I did not desire an oppositional relationship with the teachers in my study and rejected this role of expert for myself.

Thus, when working with the teachers, I resisted taking an active role as a mathematician and as a teacher educator. I experienced a great deal of conflict because of my notion of 'expert.' This was especially evident as the teachers became rooted in the narrative dimensions of mathematics. Instead of consistently offering mathematical tasks as I had done with the students I was teaching, I resisted the role within our community as a teacher of teachers. I did not want to be their teacher; I did not want to be perceived as the 'expert.' Yet, I was unhappy with the tension between the narrative and paradigmatic dimensions of knowing mathematics[16]. The notion of co-construction, for me, was problematic. I was unclear about my place within the group but was beginning to reconsider my rejection of the role of an expert. I observed many instances where Barb acted as an expert teacher and mathematician without severing her relationships with the other teachers in our group.

An Ease with Expertise

I learned from Barb how to be at ease with my expertise in mathematics. Barb proclaimed her love for mathematics during many of our group meetings without reservation: "I've never had trouble with math. I love it." I tended to hedge by including myself in generalizations about the struggle some teachers had with mathematics. After Christine had shared her discomfort with mathematics during the first group meeting, I commented:

> I think that's the nice thing about coming together, that many of us have similar experiences, myself included. Struggling with the subject and wondering what happens when we have to teach it. I think a vast majority of the people I run into that teach elementary math have that experience at some point. And so it's not uncommon.

Since I had experienced tension with my own self-identification as a mathematician, I wanted to know if the teachers viewed themselves as mathematicians. Barb described herself as "a pretty good mathematician" yet her mathematical expertise was directly connected to her ability to teach mathematics. When asked to clarify her idea of a mathematician, she suggested that she was a "teacher through and through" and that "math is just as important as the reading, as important as the writing." As a teacher with mathematical ability, she was able to share her expertise with other teachers in her school who were having difficulty teaching a particular concept. She linked her development as a mathematician with her ability to continually learn pedagogical approaches to help her students:

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[16] See p. 99 for a discussion of narrative and paradigmatic modes of knowing.

> I always find that I guess if I see a child struggling, I'm always thinking of a better way or a new way to help them grasp that concept. And so, maybe as a mathematician, I'm really always growing with that too. Rather than just saying, "I did a good job, I taught it. There's something wrong with them."

Although Barb described herself as a mathematician, she focused on her mathematical expertise in the classroom.

Denise's view of a mathematician seemed similar to my idea of an expert. She differentiated between a person who had a "higher level of knowledge" and a teacher of mathematics:

> Often math teachers are so smart and they do well in what they've done but they can't transfer it over, can't teach it to other people. It's like they know the answer, they know how they got it, but they can't explain it.

Helping students make mathematical connections through good explanations seemed important to her. She had some mathematical expertise in this area but did not consider herself to be a mathematician.

Martha's idea of a mathematician had changed "in part because of the work we had done together." She initially believed that a mathematician "thinks very linear[ly] and in the box" but now viewed a mathematician as having a "much broader understanding of math than I do." While she was hesitant to classify herself as a mathematician, she commented that she did have a certain level of mathematical understanding and, therefore, could be considered a "toddler mathematician."

Christine was hesitant about identifying herself as a mathematician. Although she did not "see myself in those terms," she suggested, "You're acting on your world, so part of your interactions on your world have to do with mathematical things. So maybe I see myself only in that category more when it comes to responsibility for financial things." Christine believed that as a teacher, "I don't know if I really consider myself in that category even yet." However, for her, the possibility of becoming a mathematician existed and was linked to pedagogical expertise in mathematics.

While many of the teachers in my study distanced themselves from mathematicians, they could identify desirable characteristics of people who were adept at doing mathematics. They described characteristics of the person, focusing on the skills of a person rather than talking about who a person was. To me, this emphasized the difference between being labelled as an expert and being identified as having expertise. I believe that being called an expert points to identity; it is something you are. Being identified as having expertise points to a characteristic; it is something you have. I am much more comfortable with describing myself as having mathematical expertise than I am with labelling myself as an expert.

Thinking of myself as having mathematical expertise would have been helpful when working with these teachers. I did not enter into our conversations with my whole being because I was hesitant to increase the distance between *I-Thou.* Yet, I observed the way Barb negotiated her place within the group by using her expertise. My own passion for mathematics was hidden from the teachers in much of this research. I could have enhanced the opportunities that emerged within our conversations by taking a more active role in consistently offering mathematical tasks to the teachers. I could have shared my expertise with them by meeting them as a mathematician and as a teacher.

When I reflect on the transcripts of our group meetings, I am aware that the teachers interacted at the beginning of each meeting by talking about pedagogical challenges they were facing in their classrooms. Initially, I thought these conversations were useful in building strong *I-Thou* relations. I have since come to understand that these conversations were replete with opportunities for the teachers to engage in mathematical tasks. For them, the tasks of the classroom were the mathematics that they needed to work on themselves because they did not have a robust understanding of these concepts. They were able to see themselves as developing mathematicians because they had worked on the mathematics. For me, this was an unexpected outcome of the conversations. I now believe that I could have participated more fully in the group conversations by sharing my mathematical expertise with them more consistently. I could have resisted their resistance to mathematize the story we were creating by offering relevant mathematical tasks to them on an ongoing basis.

Through my work with the teachers, I have come to better understand my relationship to mathematics. I believe that there is a need for mathematical expertise in this work with elementary teachers. Buber writes, “A community is built upon a living, reciprocal relationship, but the builder is the living, active center” (p. 94). I can be this builder and still sustain robust *I-Thou* relations through my expertise.

Chapter 6 – Writing New Images

Mathematics is Story

Denise: Long, long ago, before there were numbers, things were quite different and much more difficult. One day a hard-working peasant named Matthew took potatoes to the market to trade for other services. He heard that Joseph, a merchant at the market, was very fair when it came to trading. So Matthew rushed with his potatoes to make a trade with Joseph. "I have SOME potatoes to trade Joseph. Can you trade my potatoes for SOME milk?" "Yes, Matthew, I can trade with you but, tell me, what does SOME mean?" Matthew thought. "I don't know what SOME is Joseph but this is what SOME looks like." He handed a dirty burlap bag to Joseph. Joseph peered into Matthew's deep, dark bag and snapped at him by saying, "I can't see SOME. Your bag is too deep and too dark." "The potatoes are all at the bottom." "You need to show me SOME." So Matthew turned the bag upside down and SOME potatoes dropped to the ground with a thud. "Here is SOME," Matthew said with pride. Joseph laughed and said, "I traded SOME potatoes with Alexi yesterday and his SOME was much more than yours. I cannot trade SOME milk with you today. Come back tomorrow with SOME MORE potatoes." Matthew understood what Joseph wanted so he went home and collected SOME MORE potatoes. When he returned to the market the following day, he gave his SOME MORE potatoes to Joseph and received the milk in return.

Christine: That's cool. I like it.

Barb: Excellent.

[laughter]

Denise: So vocabulary.

[laughter]

Barb: Language.

Christine: S-O-M-E. I thought you were going to change it to S-U-M.

Barb: That's excellent. That would be

Christine: Wonderful.

Barb: Wonderful. That's funny. That's great.

(Fifth group meeting, October 14)

For as long as we can remember, stories have existed and helped us to remember. In ancient times, these stories were communicated orally and pictorially. Over time, stories began to be written and the study of human culture became centred on fine arts and literature. It was evident that the development of language and the creation of stories were interwoven. Egan (2004) argues that all human cultures use stories for increasing the memorability of oral language. Moreover, he claims:

> The story was perhaps the most important of all social inventions, in that it provided the bond for languaged people to tie themselves into more complex societies than extended kin-groupings and to emotionally commit themselves to shared social and cosmic stories. (p. 6)

Historians suggest that mathematics was integral to these shared cultural stories. Joseph (2000) presents evidence that ancient Egyptian, Babylonian, Chinese, and Indian cultures developed mathematical languages for interpreting their worlds. Lunar calendars were generated in order to record the passage of time, a necessity for anticipating and adapting to seasonal changes. Thus, mathematics became closely linked to cosmic stories of a society. However, mathematics was also connected to its social and economic stories. By distinguishing qualitatively and then quantitatively between *more* and *less*, counting, and recording the counts as tally marks and knotted strings, people were able to develop efficient systems of spoken numbers and written symbols. In fact, according to Joseph, written language and systems of numbers are evident and intertwined in every culture:

> No society, however small or remote, has ever lacked the basic curiosity and 'number sense' that is part of the global mathematical experience. The need to record information that gave birth to written language also brought forth a variety of number systems, each with its own strengths and peculiarities. (p. 348)

Bishop (1994) also believes that mathematics is embedded in all cultural groups and describes six fundamental mathematical activities all people engage in: counting, measuring, locating, designing, explaining, and playing. He links mathematics with language by proposing that both are part of a culture's accumulated knowledge and concludes that activities in all cultures require mathematical thinking. However, the relationship between mathematics and language is much stronger than shown through simple comparisons of similarities. Clearly, language activities are inherent in these mathematical activities as people communicate their developing knowledge of mathematics within a culture. Thus, mathematical thinking and language seem to be interdependent.

In considering the advent of the human race, Smith (1951) connects mathematics to cultural developments in early times. He writes:

> [Mathematics] appeared in art, the world's universal language; in religious mysticism, the expression of our efforts to fathom the unfathomable; and, in a less degree, in commerce, war, and the needs of the pastoral life. Each of these human interests, but especially decorative art, contributed to an appreciation of geometry; each, but especially religious mysticism and commerce, contributed to the development of number; and each lent its aid to the creation of an interest in architecture and in the science of astronomy. (p. 5)

It appears that mathematics, in its early history, was thoroughly embedded in what we would now consider to be the humanities. Indeed, *mathematics* was a term that originally described the disciplines of the quadrivium, the liberal arts comprised of arithmetic, geometry, astronomy, and music.

Currently, a fracture between the humanities and sciences exists and mathematics is placed firmly in the latter category. According to the Oxford English Dictionary, mathematics is "the science of space, number, quantity, and arrangement, whose methods involve logical reasoning and usually the use of symbolic notation, and which includes geometry, arithmetic, algebra, and analysis." As a science, mathematical writing has become preoccupied with logical reasoning and symbolic notation, as evident in this definition. Certainly, many mathematics educators and mathematicians resist the narrative quality of mathematics. Solomon and O'Neill (1998) claim that mathematical knowledge is not narrative. By arguing that cohesion in mathematical text is achieved through logical rather than temporal means, they suggest that mathematics is tenseless and timeless. They present the genre of mathematical text as defined by graphs, equations, proofs, and algorithms. However, Solomon and O'Neill seem to ignore the importance of patterns and relationships in mathematical thinking. The 'genre' of mathematical text throughout history has included narrative letters, explanations, poetry, and word problems—texts of patterned, storied thought. The genre of mathematical text referred to by Solomon and O'Neill could be viewed as the artefacts of mathematical inquiry of a particular era.

The present-day separation of mathematics from the humanities is no longer feasible. Returning to and expanding the notion that mathematics is socially constructed and negotiated, mathematics educators and researchers are promoting alternative images of mathematics that emphasize the mathematical processes of communication and connections. Teachers are beginning to understand that the development of mathematical concepts occurs in a contextual and relational manner and that this context can provide meaning. When mathematics is placed in a social and cultural context, mathematics can be viewed as a humanity. The metaphor *mathematics is story* emphasizes an encounter with the human dimensions of mathematics.

An evolving understanding of mathematics as a humanity supports the idea that mathematics can be known through cultural stories of mathematical activity. These stories resonate with our experiences of life. Thus, the metaphor *mathematics is story* embodies the characters, settings, plots, and problems of life. The intentions of stories are to show our relationship to our world. They are meant to recall and prompt thoughtful consideration of our lived lives. Stories in this narrative sense become dramatizations of mathematics and support rich, emotional, and engaging encounters with mathematics. How did the metaphor *mathematics is story* impact the teachers' views of mathematics? How are story and mathematics related?

I was interested in how this metaphor could be used to prompt the teachers' consideration of mathematics as a humanity. Through this metaphor, I sought to explicate links between two seemingly dissimilar concepts: mathematics and story. The metaphor *mathematics is language* had emerged spontaneously in our conversations and provided me with an opportunity to make a smooth transition to an explicit consideration of *mathematics is story.* During the fifth group meeting, I introduced the metaphor and asked the teachers to write reflective responses on how mathematics was like story. The metaphor *mathematics is story* was taken up in a variety of ways.

For Christine, mathematics, like stories, provided a context for meaning:

> Math is used in context to give meaning, to give it meaning, or it needs a context to have meaning. And stories require context for our understanding. Math expresses ideas, problems, solutions even questions often in symbol form, to describe the world that we know with some kind of accuracy. Stories attempt to. The accuracy is less important.

However, after briefly considering the notion of contextual meaning, she talked about the familiar story components of setting, plot, characters, conflict, and resolution. She linked some of the story elements of problem, solution, and action to story elements of mathematics. Denise also identified story elements found in mathematics and suggested that "it involves words for vocabulary, process, problem, conflict, team work, resolution". She claimed that mathematics "followed a pattern like a story. It has a beginning, middle, and end."

Similarly, Martha included the elements of a good story in her description: "[Mathematics] has several characters who play different roles in different situations. These characters are constantly getting into difficult situations. Through careful planning, creative movement and sometimes pure luck, a solution is usually found." The story pattern of beginning-middle-end and elements of characters, plot, setting, problem, and solution were evident in her account.

However, as we continued our conversation, it seemed evident that Martha's way of thinking about *mathematics is story* was connected to her experiences with story problems. When describing her approach to writing a story in mathematics, she focused on providing a situation involving subtraction within an authentic context of her own family: "Andrew knows how to count and he's starting to teach Jenny how to count. So he says, 'Pick up two toys. And she picks up three, he says, 'No, just two.' And he takes one away." This could be rewritten into a familiar story problem format that might look like this: Andrew is teaching Jenny to count. If Andrew tells Jenny to pick up two toys and Jenny picks up three toys, how many toys does Jenny have to put back? While Martha's description did not include a specific question, the question is implied and the solution is evident in Andrew's removal of one toy. This interpretation of *mathematics is story* emphasized stories *in* mathematics. In other words, for Martha, the metaphor *mathematics is story* conjured up familiar images of story problems in school mathematics. How are story problems like stories?

Stories *in* Mathematics: Relations through Story Problems

Mathematical story problems, also referred to as word problems, have a long and continuous tradition throughout the history of mathematics. In her book explicating mathematical word problems as genre, Gerofsky (2004) provides convincing evidence that story problems existed in ancient Babylonia, ancient Egypt, ancient China, ancient and medieval India, medieval Europe, the medieval Islamic world, and Renaissance Europe. Moreover, they continue to be familiar to present-day mathematics teachers throughout most of the world. Why have story problems remained deeply rooted in mathematics?

To address this question, I investigate both the purposes and features of story problems. Gerofsky's work is prominent in this chapter because she has described the genre of word problems in great detail and this kind of investigation is absent in most of the research literature. Her writing informs my thinking about what *It* might be and is fruitful when interpreting the teachers' tendency to link *mathematics is story* to *mathematics is story problems*. Story is described as having contextual meaning. Bruner (2002) presents the notion that story involves a concern for the human condition; stories resonate with people. How are stories and story problems similar? To what degree do story problems resonate with life experiences? How do story problems promote an understanding of mathematics as a cultural activity? In other words, to what extent might story problems contribute to a view of mathematics as a humanity?

According to Gerofsky (2004), the purposes of story problems have changed over 4000 years from riddles to exemplars of generality to practical problems. Referring to clay tablets from ancient Babylon dating from the period 2000 to 1600 B.C., Gerofsky argues that these ancient story problems used "unrealistic numbers, impractical situations and questions that would never arise in real-life situations" (p. 127). While some ancient Babylonian story problems seemed to refer to practical everyday situations, most were not useful.

This ambiguity is explicated by Gerofsky as she considers the relationship between story problems and orally transmitted riddles. She suggests that word problems were used as recreational examples of mathematical methods. The mathematics was embellished by the story to make it memorable. Nonsensical stories in word problems had an added benefit of encouraging responses of amazement and delight. Stories enabled problems to be remembered and orally communicated. While it appears that the stories of word problems were used to increase memorability, according to Gerofsky, the purpose of the story problem to exemplify the method persisted.

Gerofsky further argues that before the invention of algebra, repeated examples of story problems were used to show general methods for solving problems. For example, Babylonian texts contain numerous story problems that involve the area of a square. She believes that these problems were used to practice a general method for solving what we would now call quadratic equations. Thus, she proposes that ancient story problems were used to practice general solution methods through numerous repetitions.

Once algebra was invented, the use of story problems as a way of expressing generality through examples was no longer necessary or efficient. Gerofsky claims:

> Because word problems were cut loose from their necessary role in stating mathematical generality, their use had to be shored up by a sometimes spurious emphasis on the practicality and applicability. Since Renaissance times at least and certainly in our own day, we have seen many instances of the use of word problems being justified by their practicality and usefulness. (p. 131)

While she acknowledges the shifting intentions of story problems, she continues to privilege the function of story problems as exemplars of generality. She suggests that present-day story problems may be offered to pre-algebraic or novice students as repeated exemplars of generality and dismisses the current preoccupation with viewing story problems as practical.

In her earlier writing, Gerofsky (1999) describes story problems as coded and ambiguous generic forms that point to the objects of mathematics rather than to the experiences of everyday life. By considering the intertextuality of similar genres such as parables and riddles, she concludes that differences are evident in the intentions surrounding story problems. Story problems are used to practise algorithmic methods. When solved, the problem becomes disposable. In this manner, "word problems imitate and recall other word problems, not our lived lives" (p. 37). For her, story problems are intended to be disconnected from lived experiences.

The differences between story problems and stories are apparent. Gerofsky (2004) identifies the story as a feature of story problems. Implicitly recalling the story structure of beginning-middle-end, she suggests that the structure of story problems usually has three parts. These components are identified as a set-up part, an information part, and a question. However, she argues that the set-up component establishes the characters and setting of the story and is not essential to the solution of the problem. She describes this as "the only nod toward 'story' in the story problem" and suggests that "too much attention to story will distract students from the translation at hand, leading them to consider 'extraneous' factors from the story rather than concentrating on extracting variables and operations from the more mathematically salient components 2 and 3" (p. 29).

She writes, "Some word problems in my calculus textbook were difficult to approach, not because of the unfamiliarity of their subject matter but because their extreme whimsicality distracted me from approaching a solution method" (p. 6). She recalls her response to a story problem involving a person who carries a can of paint up a helical staircase only to discover that there is a hole in the can and paint has leaked out steadily during the ascent. She wonders if the paint produces a design on the stairs, if the paint drips through the rungs onto the cows below, if the person becomes angry when discovering the leakage, and if the person throws the rest of the can of paint down the stairs. Gerofsky views this distraction as problematic.

Indeed, other researchers have commented on this phenomenon. Pimm (1987) refers to a story told by Philip Roth when his father gives him a story problem to solve involving the choices a clothing dealer makes when selling an overcoat that is out of style. In this recounting, Roth is preoccupied by the story and wonders if the man buying the overcoat is laughed at, if the overcoat is needed for a fancy ball, and if last year's style looks vastly different than this year's style. Roth comments that his father is disheartened by his attention to irrelevant details and by his inability to focus on the arithmetic solution. Rather than viewing this engagement with the narrative content as problematic, Pimm writes:

> This ability to throw away the outer container of 'irrelevant' information in an essentially arithmetic task is a skill which teachers attempt to teach in schools. Yet the contexts in which these problems are embedded do carry considerable social implications for the perceived uses and usefulness of mathematics. (p. 14).

What I find interesting is that both Gerofsky and Roth have vivid memories of the context and their recall of the mathematical problem is quite strong. Although they acknowledge this as an unintelligent distraction from the mathematics, years later they remain engaged in the story.

While stories within story problems seem to resonate and increase memorability, they are not meant to ground the problem in everyday life. Thus stories are different from story problems. Gerofsky (2004) emphasizes what she views as the consistent superficiality of story problems:

> So the words in word problems do not actually refer to "real-life" objects and happenings in their full range of meanings and properties but only to those abstracted qualities which are isomorphic to the particular mathematical structure being illustrated. Most of our practical knowledge of the everyday world must be left behind when dealing with the stories of word problems. In these stories, there is no friction, no spillage, no unevenness or lack of uniformity—the contingencies of the world we know are deliberately omitted and any student who knows too much about the purported real-life subject of a word problem is

> well advised to leave that knowledge aside during math class. Word problems are not *about* the world we live in; they are about the world of mathematics. (pp. 128-129)

Story problems have been a core element of mathematical experiences. However, as Gerofsky suggests, the story component of these problems is superficial. The story created by Denise, and presented in the initial excerpt of this chapter, does not seem to be a story problem. This is a different *It*, a different genre. In what ways was it different from Gerofsky's notion of story problem? How did the teachers in my study view this alternative genre?

Mathematics *in* Stories: Relations through Literature

During the fifth group meeting, we talked about our understanding of where numbers came from. We discussed at length the role of language and communication in the development of numbers (see Chapter 5). I then asked the teachers to write a short story about the origin of numbers by placing characters into a setting and developing a story line. In a previous meeting, I had offered a few examples of what this could look like (see Appendix E for a list of the resources I used) and they had had an opportunity to observe my own interpretation of this while working in their classrooms (see Interlude 1). Most of the teachers engaged in the writing task, likely because of our previous conversations about language, communication, and connections. I chose this task because it embodied my views of mathematics as a humanity (see p. 8). Believing we learn through stories, I wanted to engage the teachers in a process of story writing and telling.

After the stories were read, the teachers agreed unanimously that Denise's story offered many possibilities for investigating mathematics. The pedagogic intention of this particular story differed from the intention of story problems. In story problems, the intention is for students to sort through the given information to determine what is relevant, to discard the extraneous information, to translate the relevant information into a mathematical question, and to calculate the correct answer. The intentions and the features of this story more closely reflected those of literature than story problems.

Martha described Denise's story as attempting "to make math more meaningful or relevant for kids, to allow new concepts to be coupled with prior knowledge or experience." Martha emphasized the significance of connecting to everyday experiences:

> And this is more like real life. You know? There aren't many times in real life, outside of teaching life, where you've got to sit down and do the old algebraic equation. It's more like I've got *some* and I need *some more*. So the way we use math on a day-to-day basis is a more general application of it as opposed to math class where it's all very "my world in a box."

Like Martha, Christine emphasized contextual meaning and claimed that this story was written "to see relationships between life and use of numbers" and "to draw out an awareness of math in so much of what we do."

The intentions expressed by the teachers in my study reflected the intentions of stories, not story problems. This story seemed thoroughly grounded in the narrative. Denise said:

> The message from the story might be that long before there were numbers, people had to communicate in different ways. The views of math in the story might be that math has a vocabulary, math has to be communicated through

> words, and the problem solving aspect and forming an answer for a reason or to meet a need.

Clearly, contextual meaning was not discarded as irrelevant.

Consistent with the intention of this story to focus on contextual meaning, its features as identified by the teachers differed from those features of story problems presented by Gerofsky. Whereas Gerofsky suggests that the set-up component of story problems is not essential for the solution of the problem and potentially distracting, the story context played a significant role in the Denise's story. Typically, stories contain elements of setting, characters, plot, problem, and solution. All of these were developed in detail within this particular story. What is the context of Denise's story? How does this context create meaning for the human condition?

The context of the story is quite familiar. It takes place "long, long ago before there were numbers" and this introduction immediately conjures up other stories of "once upon a time." The main characters are given commonplace names, names that recall Biblical times and Biblical stories.[17] The plot involves issues of fair trade, a situation that can be visualized with ease by most people. Unlike the concise set-up component of a story problem identified by Gerofsky, this story contains description and detail. The characters engage in a conversation about quantities and their emotions are evident when Joseph snaps at Matthew, when Matthew speaks with pride, and when Joseph laughs at Matthew. The context appears to be an integral part of the story.

Meaning for the human condition within stories is often created by "making the familiar and the ordinary strange again" (Bruner, 2002, p. 9). The story written by Denise seems to make the familiar strange in a number of ways that relate to the second component of Gerofsky's features of story problems. The second component found in story problems provides numeric and operational information for solving the problem. This information is not explicitly provided in Denise's story but is presented in unexpected ways.

The connection between numbers and language was familiar to the teachers because of previous conversations. For example, during one meeting, Martha told a childhood story of arguing with her sister about the meaning of *a couple.* She thought that *a couple* was two and *a few* was more than that. Her sister believed that *a few* was two because it rhymed. When Martha told this story, she remarked that she remembered it "as clear as a bell" and included the setting of the argument in her description. For Martha, there was a formal element of meaning when specific numbers were assigned to the terms *few* and *couple*.

Martha's story had resonated with Denise. However, Denise stated that *a few* meant three. Through our dialogue, we investigated the idea that the common use of this language was vague and ambiguous. Denise said, "It means something different to other people." Barb added, "Numbers mean specific things." By considering the question, *How much is some?*, Denise wrote about a familiar experience. It resonated with the teachers because it directly connected to issues emerging in our interactions. But the familiar was made strange because the story did not use numbers. When I first heard this story, I wondered why these seemingly vague words were meaningful to the characters. I resonated with the idea that numbers enabled clear communication. I understood why numbers were needed and I was drawn into a world where numbers did not exist. I was intrigued. What would life be like without numbers? The story invited me to look at my world from a different perspective.

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[17] The evocation through Biblical names of Biblical times is something I address in the next chapter.

Making the familiar strange by viewing this particular story through a mathematical lens is different from the mathematical task of solving story problems. In the case of story problems, the strange is made familiar by recognizing the mathematical elements and stripping away the details of the story. The mathematical information needed to solve a mathematical problem or perform a calculation is explicitly provided in what Gerofsky identifies as the second component of story problems. But in this story, the mathematics is not necessarily obvious; it must be attended to. Drawing on the work of David Wheeler, Phillips (1996) describes this process of attending to the tacit presence of mathematics as mathematizing. Mathematizing this particular story involves translating something that is not obviously mathematics into something that obviously is.

For me, mathematizing this story involved a creative translation that was not dependent on explicit information provided in the story. I interpreted the human concern in this story to be centred on fairness and the story did not contain enough information about how much *some* was. Therefore, mathematizing the story could involve a variety of tasks focused on measurement, rate, equivalence, and pattern recognition. It is evident that the second component of story problems identified by Gerofsky was not included.

Because a specific question was not posed, Gerofsky's third component was missing. In fact, there does not seem to be a mathematical problem in the story as shown by the reaction of the characters to Joseph's request. Matthew understood what Joseph wanted when he indicated that some more was required. This response was unexpected and I wondered how much more Matthew would need. However, these characters did not seem to grapple with this. Denise identified the importance of not seeking a correct answer. Barb and Christine said that it was valuable for students to talk about their ideas of *some* and *more* rather than to solve a particular problem. And Christine liked that it was "broader than a story problem, open-ended, not confined to one answer or one method for solving."

One of the greatest tensions for me was the lack of an identifiable mathematical problem or challenge in the story. Denise explained that "there is no problem here because the thing is that everyone in the story seems to know what *some* is. It's just us that's having the problem." Christine and Martha agreed that the reader did not know and was left on the outside. Denise said that this was her intention when she wrote the story. She had thought about adding a question at the end of the story but decided against it. Martha was concerned that the question would limit the student activity. There was a great deal of resistance to imposing one particular question on the students. I suggested that without it, it might be difficult to make the story mathematical. I wanted to understand how the teachers were thinking mathematically about the story. How did they mathematize this story?

Denise understood that *some* was a number and indicated that the story was about quantity. However, she seemed unable to elaborate on why this was mathematically important. What is the mathematical significance of *some*? Walkerdine (1988) identifies terms such as *same*, *different*, *more*, *less*, *lot,* and *little* as central to young children's acquisition of a mathematical lexicon. Of significance are the relationships of size and quantity and how these relationships are communicated in everyday contexts. The term *some* fits within her description of the lexicon and points to relationships within a set or collection of objects. An understanding of numbers rests on a recognition of number relationships. Classifying collections, comparing sets, counting, ordering, identifying patterns, and identifying one-to-one correspondence are some of the foundations of number theory. Understanding how much *some* is is of central importance to the development of early number concepts. Thus, the investigation of the quantity of *some* in the story is mathematically robust. While the teachers in the study seemed intuitively drawn to the notion of *some,* they did not seem to explicitly recognize the mathematical importance of *some* in the story.

Martha struggled with mathematizing the story. Martha suggested that "our tale is more open-ended, depending on the activity assigned. [...] It could be enjoyed just as a tale or extended further" and tended to view the story through the lens of language arts. When asked how this story was mathematical, she responded:

> It depends on the activity and what you do with it. This could be just a story in and of itself. It's fun to read [...] But you could take this and make it mathematical. You could also take this and make it more writing. So you could say, if this is descriptive, what's missing? Does your story look like my story? Or does your picture look like my picture? No? Okay, is it clear? And you could do a whole LA writing lesson on clarity and word choice.

Similarly focused on linguistic clarity, Christine suggested that "you almost need that to do anything with it mathematically." For her, communication was the primary intention of the story:

> We're writing our own version of *The Math Curse* right now and that's what's giving them trouble. How do I say this so the reader knows what I mean? When you're putting the things down. They know what they're trying to say but the rest of us don't really know what they're getting at yet because they haven't got the wording that explains what they really want to do with it. I'm not sure they know exactly what they want to do with it either. They have an idea but they haven't quite figured out how that will all work, the vocabulary to go with it.

Part of the next phase of the project involved implementing the story in a mathematics lesson. Since some of the teachers seemed to be struggling with how this could be done, I provided an opportunity for them to brainstorm some ideas during the sixth meeting. I was very interested in how the teachers would mathematize the story. Their suggestions were:

1. Have students role play and pretend that they are characters in the story. Have them explain what *some* is.
2. Have students modernize the story by rewriting it with numbers and current context.
3. Have students compare numbers of potatoes needed for a bucket/jug of milk. E.g. 7 potatoes = 1 bucket of milk. Have students graph the relationship using a line graph or pictograph. Have students make predictions from the graph.
4. Have students skip count to find out how many buckets of milk are needed.
5. Have students write subtraction sentences. E.g. if they have 16 potatoes, how many buckets of milk can they get?

6. Have students investigate overproduction scenarios when the trade equivalence changes. Provide a graph and have students interpret the pattern of economic growth or decline (supply/demand).
7. Have students consider fairness. E.g. how full does the bucket have to be? What about the size of the potatoes? Have students investigate mass, capacity, standardized quantities, and the need for standardized measurement.
8. Have students work with multiplication. E.g. 4 buckets of milk = how much potatoes? Have students investigate potato plant yields and area of field needed for crop.

All eight suggestions considered mathematical equivalencies to various degrees. However, the formats of several were reminiscent of story problems and many of the teachers seemed to default back to this familiar genre. For example, students were asked specific questions in suggestions 7 and 8 and were required to provide a subtraction sentence as an answer in suggestion 5 (although I am not sure why this would be considered a subtraction operation). Some translation might be involved in suggestions 1 and 2 as students determine what *some* is and as students substitute words with numbers but the tasks remained rooted in language arts. While many of the suggestions offered students an initial opportunity to engage in mathematics in imaginative and productive ways, after the students had determined an equivalency, the tasks constrained the potential activity of the students. Some of these teachers seemed to rely on the format of story problems in order to mathematize the story. They presented a hypothetical set-up based on an equivalency (7 potatoes = 1 bucket of milk), provided information (if you have 16 potatoes), and included a question that reinforced the need for one correct answer (how many buckets of milk can they get?). Others were able to offer students an opportunity to explore mathematical ideas of fairness, economic trade, and crop yields that were embedded in contexts of human experiences.

One interpretation of *mathematics is story* emphasizes mathematics *in* stories. In this perspective, literature is viewed through a mathematical lens. Denise commented that "most stories have math in them." Denise often combined mathematics and literature and Barb suggested that she "liked the combination of language arts and mathematics." When we explored what a mathematical story might look like, Denise admitted, "I don't know how I would have done just a pure mathematics story. Maybe I've just read too many stories of math." Throughout many of our conversations, it became clear to me that both Barb and Denise were able to mathematize literature even when the mathematics in the story was not explicitly portrayed. They seemed to encounter robust *I-Thou* relations to mathematics when they used literature in their classrooms. However, when I offered examples of mathematical stories such as *Math Curse* (Scieszka,1995) and *How Much is a Million?* (Schwartz, 1993), these were rejected by Barb, Denise, and Christine as being examples of literature, not mathematics. Literature and mathematics remained disjoint and exclusive categories for them. What kind of story was this then?

In the story created by Denise, mathematics was viewed as a language for communication and a language that described our world. This story resonated with the metaphor *mathematics is language* (see Chapter 5). The extent to which many of the teachers remained rooted in the narrative was evident in our conversations about the genre of this story. Rather than viewing the story as a story problem or as a historical story, the teachers seemed thoroughly grounded in the sub-genres of literature. What kind of literature did they view this as? What might this indicate about their image of mathematics?

A Mathematical Folktale

During our sixth meeting, I asked the teachers how they would classify their story:

> Denise: I would say, not a folktale but I would say folklore maybe? I don't know. I just think of long ago.
>
> Barb: It's a historical story though, too. It's not based on history but…that's a good question.
>
> Denise: It's kind of made up so it's more folklore, isn't it?
>
> Gladys: It's not a legend?
>
> Barb: No.
>
> Denise: No. Maybe a folktale or something that was
>
> Barb: More along that line. Folklore.
>
> Gladys: Folklore, folktale, parable, fable? I don't know.
>
> Denise: A fable would have a moral, wouldn't it?
>
> Martha: Based in history but it's fiction. Isn't that the definition of historical fiction?
>
> Barb: Yeah, that's why I was thinking of, history, historical fiction. But…
>
> Christine: I guess I always thought of historical fiction as being a little, well, closer to some specific thing
>
> Barb: Event
>
> Christine: Event or activity or
>
> Barb: And this, we don't know who exactly invented numbers.
>
> Denise: Or exactly when.
>
> Barb: Yeah exactly. I tend to think more of a folktale.

From our conversation, it was evident that the sub-genre of this story was difficult to identify. The teachers brainstormed different ideas and eventually settled on the term *folktale* to describe their story. The notion of this story as a story problem was not considered.

What are folktales? How are they related to folklore? Folktales are rooted in traditional beliefs. They are very old; their authors are unknown. Usually part of oral story-telling traditions, they are intended for all audiences. In an interview with Bill Moyer, Campbell (1988) ranks folktales as a low level type of story because they are stories that ordinary people tell for the purpose of entertainment.[18] The term *folklore* is partially derived from *lore,* which refers to the act of teaching and has similar etymological roots as *learn*. When a story is referred to as *folklore*, this implies that a lesson is to be learned. Campbell's assumed hierarchy of stories in literature is compromised, as the story's purpose is no longer solely for entertainment. How did the teachers understand *folktale* and *folklore*?

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[18] Campbell distinguishes folktales from myths, a high level type of story whose purpose is for spiritual instruction.

The descriptions given by the teachers matched many of the characteristics of folktales. Christine suggested that the story took place long, long ago. Its characters, while members of a particular community or part of the world, resonated with the teachers. Its setting was long ago, but Martha commented that "this could just as easy have been in rural Alberta in the late 1800s or early 1900s." The familiar context of the story was viewed by Martha as a strength because "that leaves the door open, more open for kids to be able to hang it on whatever experience or background knowledge that they have." The implication was that this story resonated with people because it was reminiscent of everyday life and common experiences.

Although the story seemed ordinary, Martha claimed that "it's fiction as opposed to being a legend that is supposedly based in reality, fact. That this is not that." She resisted the notion that the story might be an account of historical events. Along with most of the other teachers, she remained rooted in the narrative.

Both Martha and Denise mentioned that this particular story reminded them of stories told and retold by many generations of grandparents. Was the purpose to entertain? Denise proposed that the word *some* was a constant throughout the story, a common thread that indicated an idea, a lesson, or a learning. She stated that it was not a moral but something was learned. The story was not told merely to entertain. Its purpose did not exactly reflect that of a folktale yet there was a lack of specificity and an undeveloped lesson that limited the impact of the story.

Since the teachers continued to envision the story as a folktale, I wondered how this perspective was influencing their image of mathematics. To me, it seemed that they mathematized the story in a variety of ways. Martha expressed this by talking about the "crossover in curriculum" where "you've got one story and you can do LA stuff with it, you can do social studies stuff with it, you could do math stuff with it, you could do science stuff with it if it's with plants. You know, there's all kinds of stuff you can do there." The story opened up many possibilities for student tasks. However, I was concerned that the folktale nature of the story would encourage teachers to attend to areas in the curriculum they felt most comfortable with. I felt strongly that mathematizing the story would be difficult for many teachers because the mathematics was not explicitly emphasized. I investigated this tension by asking the teachers for their interpretation of the image of mathematics being portrayed in the story.

Mathematics *and* Stories

Story problems are prevalent within mathematics. Indeed, they are often the sole 'narrative' genre experienced by students in school mathematics. However, story problems are not narrative in the sense that they resonate with human experiences. What appears to be the sole intention of story problems is the practice of a correct method in order to get the correct answer. But this may reflect an anachronistic view influenced by a heritage steeped in tradition or by the "styles of communication which are *conventionally* considered appropriate to particular disciplines" (Pimm, 1987, p. 12, italics in original). Perhaps story problems are not the only genre that prompts mathematical knowing.

Conceivably the persistence of the story component in word problems points to a human need for context and story. Mathematics *is* about the world we live in. Clearly, an alternative narrative genre may offer expanded possibilities for stimulating mathematical knowing.

I experience two tensions when I compare stories to story problems: story problems do not recall lived experiences and story problems refer to abstract mathematical qualities. Indeed, if this is so, then why are fragments of story included in the genre of story problems? Certainly there are more efficient ways of practicing algorithmic methods. And if story problems are "typically offered to students for quick translation into symbolic form, correct solution by familiar methods as an exercise, and immediate disposal" (Gerofsky, 2004, p. 153), is the sole purpose of the story to confuse and distract the problem solver? This seems illogical.

What, then, is the purpose of the story? When solving story problems, the abstract qualities of mathematics are implicitly contrasted with the concrete context of story and found to be more desirable. They are retained while the irrelevant details of the story are discarded.

The privileging of the abstract qualities of mathematics draws on a Piagetian perspective where coming to know mathematically is viewed as a linear progression from concrete experience to abstract understanding. Donaldson (1978) challenges Piagetian theories by suggesting that pre-school children can reason in sophisticated ways when tasks are presented in rich contexts that make 'human sense'. She writes:

> So long as our thinking is sustained by this kind of human sense, and so long and the conclusion to which the reasoning leads is not in conflict with something which we know or believe or want to believe, we tend to have no difficulty. Thus even pre-school children can frequently reason well about the events in the stories they hear. However, when we move beyond the bounds of human sense there is a dramatic difference. (p. 76)

Instead of contrasting concrete and abstract qualities of experience, she distinguishes between embedded and disembedded thinking where disembedded thinking is "prised out of the old primitive matrix within which originally all our thinking is contained" (p. 76). According to Gerofsky's explication of word problems as genre, solving story problems involves disembedded thinking. The story component, a possible opportunity to encourage embedded thinking, is set aside.

For me, a fruitful investigation of knowing mathematics includes a movement between particulars and generalities. Davis and Hersh (1981) distinguish between abstraction and generalization. These researchers present two related understandings of abstraction: abstraction as idealization and abstraction as extraction. Abstraction can be viewed as making the real world of experiences an ideal world of mathematical objects. It can also be viewed as stripping away aspects of thinking that are inessential. The process of abstraction can occur with very young children when they use numbers and symbols to represent and communicate their experiences. Generalizations are stronger than abstractions because they consolidate information. While the general includes aspects of the particular, moving from the particular to the general may occasion "a dramatic refocusing of interest and a reorientation as to what is significant" (p. 135). Thus, the process of knowing mathematically oscillates between the general and the particular.

The concern that a preoccupation with the story may divert attention away from the mathematics begs consideration. Pimm (1992) suggests that seeing the general in the particular is fundamental to mathematics, yet calls on teachers to "avert the destruction of meaning, of connection and any sense of personal involvement which characterizes more than caricatures the effect of much exposure to school mathematics" (p. 95). This implies that generality and embedded learning are interrelated. Indeed, Donaldson (1978) reaches this conclusion when she states, "The paradoxical fact is that disembedded thinking, although by definition it calls for the ability to stand back from life, yields its greatest riches when it is conjoined with doing" (p. 83). The inference is that immersion in the relational and contextual aspects of story supports, and perhaps enables, mathematical knowing.

While Gerofsky identifies story problems as privileging disembedded knowing, she emphasizes the importance of story in coming to know mathematics. In the conclusion of her analysis of the genre of story problems, Gerofsky investigates possibilities for pushing the genre boundaries to include unconventional forms. Many of these alternatives are grounded in narrative. For example, she plays with forms of story problems written in verse and poetry, she specifically considers word problems containing emotive prose, and she investigates various answers in the form of drawings, poems, and music. When presenting these unconventional story problems to elementary and secondary teachers, she found that many were frustrated and confused. Some were unable to focus on the mathematics because they relied on the conventional format and were unable to anticipate their role in the problem-solving process. While a few teachers acknowledged that the novelty of the problems stimulated their thinking, they did not classify these problems as story problems. The features of story problems that they had come to expect were not as apparent in these forms.

Gerofsky seems to acknowledge the importance of blending story and mathematics. Yet she is convinced that this can occur within a genre that has a long tradition of story superficiality. Linked strongly to problem solving, story problems are not meant to resonate with a particular context. In fact, competency in solving story problems involves stripping away the context. The primary purpose of including story problems in mathematics is to provide practice in problem solving. *Mathematics is story problem* focuses on getting the correct answer by using one correct method. Practice is required and a competency for translating words into symbols is necessary.

This is not what I intended by introducing the metaphor *mathematics is story* to the teachers in my study. Rather this represents stories *in* mathematics. Bruner (2002) writes, "Great narrative is an invitation to problem finding, not a lesson in problem solving. It is deeply about plight, about the road rather than about the inn to which it leads" (p. 20).

A view of mathematics as disconnected from human experience is implicit in the genre of story problems. This relationship with mathematics seems to be exemplified by Buber's notion of *I-It* except when the story engages the reader's attention and emotions. Recreational mathematics, enhanced by the novelty of the story, can encourage an *I-Thou* relationship with mathematics. However, this can be difficult to promote within a genre of story problems used in school settings. Rather than "trying new contexts for old forms" (Gerofsky, 2004, p. 153) and forcing dissonance within the genre of story problems, I suggest that a different genre of mathematical text is needed.

I claim that the story presented in the initial excerpt of this chapter is not a story problem but an example of an alternative genre of *mathematics is story*. If this is so, then what views of mathematics does this story promote?

Logos as Discourse and Reason: A Historical Turn

What did this story tell our students about the nature of mathematics? I posed this question at the end of our sixth meeting. Barb began the conversation:

> Math was an idea first. And that we use numbers to express ideas. And, so I thought this really fit in with our story. That numbers are an expression of ideas, numbers represent the idea of *some* and *some more* in our story, and that's why they're invented. And it's also this idea that numbers are an idea is key to problem solving. So that if you understand that addition is adding more, the idea of that, then the numbers just express the idea.

She added that mathematicians like Fibonacci began creating mathematics because they had an idea first. According to Barb, coming to know mathematics involved understanding the idea of what the numbers represented. Her conclusion was supported by our earlier conversations about the development of zero (see Chapter 5). The story, referred to by her as *our* story, resonated because it presented an idea of quantity.

Martha believed:

> Numbers are a necessary part of our lives. They are helpful and make life easier, than having to say *some* and *some more.* We can use numbers that makes life easier. And that numbers are a good thing [...] For a long time, I didn't think they were a good thing. Until I began to understand math better, they were a scary thing.

She linked her process of coming to know mathematics to understanding numbers. She stated, "Math has evolved out of the necessity to clearly communicate [...] To be understood is important."

Mathematics is story was interpreted by most of the teachers within a context of literature and language. By identifying the story as a folktale, the teachers emphasized the importance of the stories of ordinary people. While the context was familiar, the story was not set within a specific historical place and time. Yet in Barb's description of the image of mathematics in the story, connections to ideas of human invention and culture were evident. Relations to *It* were becoming stronger as we talked about the human agency in mathematics.

My notion of *mathematics is story* seeks to find a middle ground between story and mathematics. This space involves four dimensions: the history of the discipline over time, the history of the development of mathematical concepts, the history of mathematician's lives, and the history of personal experiences of mathematics. These are stories about the history of mathematics, as Changeux and Connes (1995) suggest:

> Mathematics is taught in school as a coherent set of prepositions, theorems, axioms. One forgets that these have appeared successively in the course of the history of mathematics and of human society—in short, that they are cultural objects subject to evolution. Putting mathematical objects back in historical perspective "secularizes" them, shows them to be more contingent than they appear. One comes to see that theories succeed one another, and that some of them manage to cast new light without invalidating their predecessors. (p. 18)

For me, a fruitful way of thinking about this middle space between mathematics and story is through an explication of the term *logos*. Derived from a Greek word, *logos* refers both to language dimensions of word, speech, and discourse and to mathematical dimensions of reason and ratio. How can mathematics be viewed as both logos as discourse and logos as reason and ratio? I investigate this middle ground through the metaphor *mathematics is historical story* in the next chapter.

Chapter 7 – Creating a Parable

Mathematics is History

Gladys: At the beginning of this project, we talked about the history of math. And Denise has brought up the fact that [the illustrator for our book] wants more information about the history of numbers. To what extent do you think this is a history of math story?

Barb: Oh, it was way more than that. Way more.

Christine: I think it's only the wording, the wording and the trading part that is. You could still do [it], they're still trading for services. [...] So it's not that it doesn't still happen. It's just maybe rarer. In other parts of the world, it does still happen. So I don't think it has to be limited to that.

Barb: Oh, no. I feel it was way more than that. It was getting the children thinking about why they use numbers and what do numbers represent?

Martha: And ancient numbers weren't always there.

Barb: But also that we use those words. It's even relevant today. We use those words some, a few, a couple. *So it's not just the history of math. It shows you the difference between that kind of language and specific language using numbers. So it's not just the history of math. And as I said, it could be an economics, trading. It's just so open-ended, the story. So, to me, it's way more than the history of math.*

[...]

Barb: I think it is important, the history, for us to introduce it as a historical story. Because it gets children really thinking about what do these numbers really mean? [...] And it's math communication. Communicating and thinking mathematically.

(Eighth group meeting, December 2)

As presented in Chapter 6, the teachers initially identified the story created by Denise as a folktale. They were rooted firmly in a narrative mode of thinking. For me, this was problematic because I was interested in finding a bridge between story and mathematics. I believed that a consideration of the history of mathematics would encourage teachers to focus more on the mathematics. In other words, I wanted the teachers to think both narratively and mathematically about the story.

Bruner (1986) distinguishes between two ways of knowing: *paradigmatic* and *narrative*. The paradigmatic mode emphasizes general causes, uses procedures to verify empirical truth, and is driven by logically generated hypotheses. Paradigmatic arguments strive to identify universal truth. For Bruner, the paradigmatic mode "attempts to fulfill the ideal of a formal, mathematical system of description and explanation. It employs categorization or conceptualization and the operations by which categories are established, instantiated, idealized, and related one to the other to form a system" (p. 12).

By seeing the general in the particular, the paradigmatic way of knowing seeks further and further abstraction from time and place. Mathematics is traditionally regarded as archetypal paradigmatic thought. The sole focus of paradigmatic ways of knowing is on relations to *It*.

Prior to the eighth group meeting, each teacher had had the opportunity to implement the story[19] in her classroom while another teacher from the group observed the lesson. Our conversations during this meeting focused on descriptions of their impressions of this process and their rationales for their choices.

Most of the teachers chose to offer story problems to their students. For example, in Denise's lesson, students were given a diagram of a field containing nine hills of potatoes, told that each hill yielded eight potatoes, and asked to calculate the number of potatoes that could be grown in the field. These choices exemplified a focus on the paradigmatic mode of knowing mathematics.

By contrast, Bruner believes that the narrative mode differs considerably from paradigmatic thought. Narrative ways of knowing focus on particular experiences, establish verisimilitude, and are driven by connections of lifelikeness. Describing the narrative mode, he writes, "It deals in human or human-like intention and action and the vicissitudes and consequences that mark their course. It strives to put its timeless miracles into the particulars of experience, and to locate the experience in time and place" (p. 13). Narrative thought is rooted in particular yet resonant experiences. Narrative ways of knowing emphasize *I-Thou* relations.

Although Denise mathematized the story by presenting her students with a paradigmatic task that involved solving a story problem, finding equivalencies, and constructing T-charts,[20] she was particularly interested in how the students experienced the story. She decided to have them illustrate it. Her initial paradigmatic approach to the mathematics shifted to the narrative mode as she focused on how the students were imagining the narrative details of the story:

> And I wanted to relate it back to [the context], because not much focus was done on Matthew and Mary's[21] field. So I wanted to take the story to their field and to show their hills of potatoes and things like that. And we talked lots about how potatoes grow and things. And I think there's a lot of experience out of that. You could see on one of their pictures, they drew all these fields. Which was kind of neat because they brought that back themselves. And like I said, I chose to illustrate the story because I wanted to see what they would remember.

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[19] An interesting shift occurred after the teachers had implemented the story in their classrooms: They no longer referred to the story as Denise's story but took joint ownership of this creation. My references in this chapter to *their story* reflect this change in perception.

[20] Resembling the letter T, a T-chart (or T-table) is used to organize, display and record data, patterns, or functions. Conventionally, input values are listed on the left, output values on the right.

[21] During the seventh group meeting, I had noticed that the story characters were male. When I mentioned my observation, several of the teachers remarked that this was historically correct. In other words, they believed that women had no role in trade and it was evident that long, long ago they had no voice. I challenged them by suggesting that an implicit message of this story was that women were not involved in mathematics. All teachers agreed that they wanted to portray women engaged in mathematics but felt limited by their understanding of history. When I offered the idea that while there were few recorded stories of female mathematicians, we did, in fact, have glimmers of their work through the male story, the 'his-story,' changes in the story were made to include a character named Mary.

The two tasks that Denise presented to her students, one of solving a story problem and the other of illustrating the story, demonstrate the two different modes of knowing mathematics. While Bruner views these two modes of thought as distinct from one another and dismisses claims that the paradigmatic mode is an abstraction of the narrative mode, he suggests that both are necessary. He claims, "The two (though complementary) are irreducible to one another. Efforts to reduce one mode to the other or to ignore one at the expense of the other inevitably fail to capture the rich diversity of thought" (p. 11).

I was interested in bringing these two modes of thought together and believed that this could be done through the study of the history of mathematics. Historical stories could involve both the paradigmatic and the narrative, thus forming a strong triad of *I-Thou-It* relations. I presented the teachers with the metaphor, *mathematics is story*, as a means of stimulating their consideration of both modes of knowing mathematics.

Since mathematics and story are generally viewed as oppositional, the metaphor, *mathematics is story*, appears to be an oxymoron and, therefore, might have been challenging for the teachers in the study. My notion of *mathematics is story* considers how the history of mathematics can be used to focus attention on the similarities between these two seemingly contradictory concepts. In other words, if paradigmatic and narrative ways of knowing are seen as dichotomous, then I am seeking to bridge the gulf between them through historical stories. However, bridging this gap can involve two paths: beginning with the history and moving toward the narrative or beginning with the story and moving toward the history. Clearly, these two approaches may not meet or the bridge may fall short. I return to this later in this chapter.

While many of the teachers included paradigmatic tasks of solving story problems, most became preoccupied with the narrative dimensions of the students' activity. Denise was surprised that "a lot of them went modern day even though we said long, long ago before there were numbers." Martha noticed that some of her students thought the story took place in pioneer times. Christine had also asked her students about how they were viewing the story and found it odd when one of them pictured the characters in white and brown. She suggested that this particular student could not form an image of the setting of the story.

It became very important to the teachers for their students to appreciate the historical setting of the story. Denise commented:

> Maybe we do need to make sure that we go back to the garb of ancient Greece and Egypt times where they're "before there were numbers" so that the kids could get a sense of how far back this was.[22]

The teachers' focus on the particularity of the images points to their preoccupation with the narrative dimension of the story. What happens when we become rooted in the narrative dimensions of mathematical stories? How are we then to turn to the paradigmatic mode of thinking? In what ways do these two modes of thought interact?

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[22] Denise's suggestion to portray the development of numbers as Greek or Egyptian was historically incorrect, even though we had explored primitive uses of tally marks and counting systems at the beginning of our study. Thus, her relationship to *It* did not appear robust as the fictional dimension of the narrative was emphasized. I return to this issue of historical inaccuracy in Chapter 8.

A reconsideration of the history of mathematics emerged in our conversations and was motivated by the images produced by Denise's students. Expanding this thread and referring to the illustrator's request for particular details, I offered these teachers an opportunity to reflect on the role of history in their story. To what extent was this tale a historical story?

A Historical Story

Typically, a historical story is set in the past and is about a specific event or person. While the details of a story may be embellishments of the truth, the focus is on recorded facts. In mathematics, historical stories have been written about the lives of 'mathematicians' such as Pythagoras, Hypatia, Archemedes, Germain, and Ramanujan, to name just a few. Indeed, in my own work with mathematics and literature, I have written stories about Euler, Descartes, Al-Khwarismi, and Gauss. My approach to creating historical stories in mathematics highlights the paradigmatic dimensions of the story that relate to a formal, mathematical system. Narrative details are added to ground the event, invention, or mathematician in a historical context (see Interlude 1 and Appendix C). The story created by the teachers was not biographical and did not describe an aspect of the life of a famous mathematician. It neither emphasized a particular mathematical event, nor was it set in a specific time or place. Clearly, it was not a typical historical story.

Christine seemed to recognize that their story was not historical. She commented that it was not about a specific past event; directly bartering or trading for services happened occasionally in the present day. Similarly, Martha focused on connections of the story to everyday events. For these two teachers, there seemed to be a time-transcending quality to the story that was rooted in the *I-Thou* relations of narrative.

Barb believed their story was "way more" than a historical story because it addressed philosophical issues involving the reasons for using numbers (and perhaps consequently providing a tacit rationale for their resistance to the paradigmatic nature of the history of mathematics). According to her, the story investigated what numbers represented and prompted thought about the meaning of numbers and about mathematics as communication.

Their story was not considered "true" in the sense of historical accuracy and factual information. Although Christine believed that mathematics expressed ideas, problems, solutions, and questions in symbolic form to accurately describe the world, she acknowledged that "stories attempt to describe both fact and fiction in a memorable fashion. The accuracy is less important." This fusion of truth and fiction is reflected in the contemporary notion of *faction*, evident for instance in cinematic documentaries that merge historical facts with fabricated details. Here, however, the paradigmatic aspects of the history of mathematics were set aside in favour of the narrative.

I had expected the teachers to encounter some tension between reality and fiction, between truth and imagination. However, this did not arise in our interactions. Throughout our conversations during the fifth, sixth, and seventh group meetings, the teachers had resisted my efforts to encourage them to depict the origins of numbers accurately in a historically correct setting. Until the eighth meeting, historical accuracy was not seriously considered. Even during the eighth meeting, it seemed that the teachers wanted to retain the folktale quality of their story but were challenged because their students were inventing a historically incorrect setting. The teachers continued to reject the categorization of the story as 'historical fiction'. Perhaps they feared that the significance of their story would be obscured if they focused on issues and questions of historical fact. In their story, mathematical "truths" involved philosophical ideas not factual data.

If their story was not true in a historical sense, what made it true for these teachers? Bateson (1994) refers to a story as coming to stand for "some renascent truth" (p. 11). Renascent truth can be understood as something that resonates with us, something that reminds us of what we already understand but are encountering in a new way. Stories do not represent 'Truth' in a platonic sense. Rather, their truths communicate meaningful relationships. In this way, stories bring forth relations of *I-Thou*. Bateson writes:

> Wherever a story comes from, whether it is a familiar myth or a private memory, the retelling exemplifies the making of a connection from one pattern to another: a potential translation in which narrative becomes parable and the once upon a time comes to stand for some renascent truth. This approach applies to all the incident of everyday life [...] Our species thinks in metaphors and learns through stories. (p. 11)

Renascent truth seems to be connected to philosophical ponderings. Like Barb, Denise emphasized the philosophical aspects of the story. By adding a title, she deliberately invited her students to consider the question: *How much is some?.* Her reason for choosing this title included an explicit identification of the story as mathematical: "It fits the math right into the title. Because I think if I was going to look for a math story, and I saw that, I would think, 'Oh, that's a math story.'" Moreover, for Denise, the question clarified the mathematical purpose of the story, which was to provoke philosophical thought about the historical origin of numbers. Hence, the story was viewed as more than a history of mathematics story.

The term *renascent* refers to something that is being born again. Linked to the term *renaissance,* it conjures up images of revival and a refocus on humanity. A consideration of *mathematics is story* can provoke dissonance between traditional and reformed views of mathematical truths. When the two concepts of mathematics and story are set side-by-side, the systematic collection of facts and procedures can be compared to the human conversation and creation of mathematics. Bruner (2002) writes, "We know in our bones that stories are *made*, not *found* in the world" (p. 22, italics in original). By beginning the story with *long, long ago*, perhaps "the once upon a time comes to stand for some renascent truth" (Bateson, 1994, p. 11). The teachers valued this narrative quality of the story.

The teachers did not want the story set in the present time and chose a biblical era. Why was this seen as important? I was perplexed by the biblical quality of the story, particularly evident in the Denise's choice of character names. During the sixth group meeting, I drew her attention to her selection:

> Gladys: Did you realize you picked biblical names?
>
> [laughter]
>
> Denise: I do actually realize that.
>
> [...]
>
> Denise: Do we want to play with names or is that good?

> Barb: No, I think so.
>
> Gladys: Do you want to put more multicultural names in it? Less western names? [...] I don't know. You want to keep it back in the period.
>
> Barb: I was going to say, this to me reminds me more of Biblical times. You've got the dirty burlap sack. It seems very biblical. So, I don't see anything wrong with that. As long as the biblical names go with the people and place.

Both Denise and Barb resisted renaming the characters, even when I offered historical evidence for an alternative place and time.

I began to investigate reasons for the biblical references. Bateson suggests that when a story resonates, there is a translation from simple narrative to parable. Traditionally, parables are related to a biblical context. I wondered if the qualities of parables were cued by the biblical tone of the story.

Story can be described as "a narrative of real or, more usually, fictitious events, designed for the entertainment of the hearer or reader" (OED, 1989). The purpose of stories is to entertain; the purpose of parables is to educate. Referring to Kierkegaard's parables, Oden (1978) writes, "Anyone who lives with these parables for a while experiences both their power and their beauty. Soon you realize that it is not you who are interpreting the parable but the parable that is interpreting you" (p. ix). Parables are teaching stories.

One difficulty with the metaphor of *mathematics is story* is that the intention of school mathematics is not to entertain, despite the increasing use of 'fun' as an articulated and explicit goal. By reframing the story as a parable, the educational intention is emphasized. During the seventh group meeting, I offered the possibility that their story might be a parable. This seemed to resonate with the teachers as we investigated the relationship between philosophy and parable.

A Mathematical Parable: Where Paradigmatic and Narrative Worlds Meet

When presented with the notion that their story might be considered a parable, the teachers immediately pondered possible 'morals' or 'lessons' of their story. Morals or lessons are common features of parables, as explicated by Dodd (1961) when he refers to the fictitious allegories explicating moral or spiritual relations presented in the New Testament. The teachers in this study grappled with these features of parables. Denise suggested that the term *some* provided a constant and common thread throughout the story and this might provide insight into a possible lesson:

> Denise: It's not a moral but
>
> Martha: It's a lesson. Or
>
> Denise: Or an idea, a learning, a...I don't know how to describe that. It's not a moral.
>
> Christine: No, it's
>
> Denise: It's something learned.
>
> Christine: Or something gained through experience.

These teachers were reluctant to identify a 'moral' of the story but the story did seem to contain a lesson for them. Denise asked for clarification of the characteristics of parables. Christine provided a biblical example of a farmer sowing seed but wondered if all parables concluded with a statement that summarized the meaning of the story.[23] Her search for an 'exact' parabolic statement at the conclusion of their story was unsuccessful.

Rather than expecting to find such a statement, Dodd (1961) suggests that the meaning of most parables is not obvious; parables are meant to provoke thoughtfulness. He writes:

> At its simplest a parable is a metaphor or simile drawn from nature or common life, arresting the hearer by its vividness or strangeness, and leaving the mind in sufficient doubt about its precise application to tease it into active thought." (p. 5)

Indeed, according to Dodd, Jesus often introduced a parable by asking a question of his listeners: "What do you think?" He comments:

> The parable has the character of an argument, in that it entices the hearer to a judgment upon the situation depicted, and then challenges [her or] him, directly or by implication, to apply that judgment to the matter in hand. (p. 13)

Like Dodd, all teachers identified an important feature of their story as leaving the meaning open for discussion and interpretation. Martha suggested that in their story, "there's no lesson like a right or wrong lesson or a moral lesson, as we call it." Christine proposed that their story contained a concept, a kind of lesson about trade. Still unsure of the characteristics of parables, Martha asked, "Does a parable leave the reader asking questions?" The other teachers expressed agreement, whereupon Martha asked, "Is that what we're doing here? Wanting kids to ask and provoke the discussion about what is *some*? And what is *some more*?" This seemed to resonate with the teachers.

I decided to seek the teachers' reactions to Gerofsky's (1999) perspective on parables. She writes:

> Unlike word problems in mathematics education, parables, koans and other teaching tales demand no solution, and often ask no direct question. They are not meant to be solved, and generally present an insoluble, paradoxical dilemma. Teaching with parables and other teaching tales traditionally involves a discussion of the contingencies of the story, of the problems of human life that relate to the story, of the sources of its paradox. Teaching tales are certainly not meant to be disposable exercises. They are made to be held onto, to irritate, to resonate. A good parable will inspire contemplation, and will be recalled in times of difficulty as a way of trying to make sense of a seeming *impasse*. (pp. 39-40, italics in original)

When I presented this paragraph to the teachers, Denise immediately replied, "That's exactly it. This isn't meant to be solved, it's a question, a thought." Although Gerofsky's explication of a parable was thought provoking, the other teachers were hesitant in agreeing that it described the story they had written. It appeared that they remained rooted in the narrative mode of thinking. How did thinking about their story as a parable facilitate a paradigmatic turn? How did this story inspire mathematical contemplation?

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[23] Dodd argues that morals appearing in the New Testament are inconsistent across parables in the synoptic gospels and that these applications were not part of earlier traditions but supplied by church authorities representing the exegesis of a particular historical era.

Calvert (2002) describes the mathematical derivation of *parable*:

> *Parabola* is, of course, from Greek, and refers to a particular plane curve. The word parabolh means a "comparison," literally "a throwing beside." The accent is on the last syllable. It is the same word as *parable*, but Pythagoras gave it a technical mathematical meaning as the fundamental operation in his method of "application of areas," a geometrical substitute for what we would now call algebra. When the curves known as conic sections were described by Apollonius, the classification of the curves rested on a certain comparison of areas. With the parabola, the two areas were equal, so the curve was so-named. With the hyperbola, an area was an excess, uperbolh, while with the ellipse, it was a lack, elliyh. (p. 1, italics in original)

Indeed, the mathematical meaning of parable is evident in its literary application. Parabolas have two equal areas. Parabolic comparisons highlight similarities between two concepts and parabolic "equality" is derived from the resonance of the metaphor. Parables do not fall short as elliptic tales might. In Eco's (1994) view, "A story may be more or less quick–that is to say, more or less elliptic– but how elliptic it may be is determined by the sort of reader it is addressed to" (p. 6). The extent to which details are lacking depends on the intended audience; however, the story is still quick. Unlike elliptical stories portrayed by Eco and stories greatly exaggerated and elaborated in hyperbolic tales, parables are neither cryptic nor larger-than-life. Like Kipling's (1996) collection of short stories, parables are 'just so' stories. They are seen to be exactly right:

> The typical parable, whether it be a simple metaphor, or a more elaborate similitude, or a full-length story, presents one single point of comparison. The details are not intended to have independent significance. (Dodd, 1961, p. 7)

The notion that parabolas have two equal parts indicates a sense of truth; parabolas are 'true' if their two areas are equal. Where does a parable get its truth? Parabolic truth depends on resonance with everyday experience. Donahue (1988) asks the question, "How does a parable mean?" and suggests that while "the realism of the parables tells us that the raw material of Jesus' language was the everyday world of nature and human activity," (p. 13) vividness and strangeness must be retained "whereby people who hear that things are not the way they seem to be may experience what they have not dared to hope" (p. 17). What makes a parable unique for teaching is the timeless quality that continues to invite alternative interpretations. Perrin (2003) writes:

> The parable stands in its own right as metaphor, open-ended, capable of transference into a hundred other situations than that of Jesus and his hearers. In their realism the parables show us a world we recognize as our own, but then they suddenly turn that world upside down. (p. 42)

Making the familiar strange is a narrative characteristic (see p. 94). Bateson's notion of renascent truth seems appropriate in this context and points to the grounding of parables in narrative ways of thinking. In what ways might parables be considered mathematical stories?

In order to turn the conversation back to a paradigmatic mode of thinking, I asked the teachers to reflect on the *mathematical* value of the lesson that they implemented.

Initially, Denise identified student activities of constructing graphs and T-charts and performing multiplication operations and Barb extended the list: "There's the number concept, there's division, there's measurement, mass, capacity. There is data management, making graphs. There's problem solving, there's patterning. There was tons [of mathematics]."

Christine mentioned that thinking about mathematics was evident and Denise commented that freedom to think was important:

> The freedom of coming up with their own ideas of what some was and showing their confidence by giving a good argument and saying, "Well, it's not fair but this milk weighs more than these potatoes." They have the freedom to speak up for themselves, especially the girls.

She highlighted the mathematical significance of argument and justification.

While the other teachers emphasized the value of mathematical processes, Martha felt that the story was powerful because it integrated language arts, mathematics, and health:

> [The students and I] were getting into almost an ethical discussion and the health curriculum and how everything was so intertwined. I found it really hard when I was planning to pull out, because you had said make sure that it's a math lesson, and I really struggled with that. Because it's an LA lesson, it's a math lesson, it's a health lesson. It's everything truly embedded together and to have to hang out one label on it, I struggled with [that].

Martha's struggle to separate the mathematical content from an ethical discussion was similar to the difficulty Barb experienced when trying to focus solely on one mathematical concept. On reflection, Barb recognized that this might not be problematic because "having this conversation with you, I'm thinking isn't that great because that's what math is all about. Math can be open-ended." The focus on open-endedness and ethical pondering point towards to the parabolic dimensions of this story.

Prior to teaching her lesson, the notion that mathematics could be open-ended was new to Christine. Her experience of mathematics involved getting *the* correct answer:

> I was thinking that was all I ever thought it was. Or that is was closed. There's always one solution, one solution. As a kid, that's all and so I'd get so frustrated when I couldn't get that solution or that one answer. You start to build this lack of confidence in the whole process if you can't nail that stupid number. Or whatever it is, right? But if you leave it so that they learn that it isn't always going to be the same solution and that it's not always the same answer, there's multiple ways to get to it, you're probably leaving kids with some level of, "Gee, I'm not so dumb after all."

When she implemented the lesson, she admitted that she wasn't totally comfortable with the open-endedness of it. Yet she recognized the value of this experience for her students. She was willing to practice being comfortable with the open-endedness because she was doing it for a purpose that involved increasing student confidence in mathematics.

Beginning from the narrative and working toward the paradigmatic seemed to offer these teachers an opportunity to reconsider the nature of mathematics. Rather than viewing mathematics solely as absolute, Christine was able to reframe how she perceived errors in mathematics. As she began to value open-ended experiences, she recognized that her students could set aside their preoccupation with getting the correct answer and could pose creative alternatives for mathematical problems. Relationships to *It* could be different. No longer was it essential that the students and teacher communicate through quizzing, explaining the right algorithm, and correcting mistakes (Alrø & Skovsmose, 2002). Rather they could move from certainty to curiosity. Mathematics could embrace human experiences. *It* could become a *Thou.* Moving from the narrative mode of thinking toward the paradigmatic does indeed offer an alternative treatment of errors, as presented by Bruner (2002):

> Narrative is a recounting of human plans gone off track, expectations gone awry. It is a way to domesticate human error and surprise. It conventionalizes the common forms of human mishap into genres—comedy, tragedy, romance, irony, or whatever format may lessen the sting of our fortuity. Stories reassert a kind of conventional wisdom about what can be expected, even (or especially) what can be expected to go wrong and what might be done to restore or cope with the situation. (p. 31)

As I began to better understand how the teachers were viewing the value of writing this type of story for a mathematics classroom, I reconsidered my view that teachers may be trying to escape mathematics through the narrative. My approach to writing historical stories was to begin with the paradigmatic and move towards the narrative way of knowing (see Interlude 1). The teachers were beginning from the narrative and were now with my pressure attempting to move toward the paradigmatic. This produced tensions because I tended to turn mathematics into a narrative rather than mathematize the story, the approach favoured by the teachers. Clearly, our approaches were not meeting halfway.

The relationship between the narrative and paradigmatic ways of thinking need not be viewed as oppositional as Bruner would have us believe. Tahta (1993), quoting the work of Michel Serres, writes: “Let us not forget that Leibniz, proto-inventor of the new science, said in time and against his time that one should listen to old wives’ tales” (p. 3). Moreover, Wheeler (1993), grappling with Serres’ ideas, concludes that the two modes of thinking do not conflict:

> First, Serres sends some clear messages about the history of science and mathematics: for example, that myths are not fictitious stories about false gods but genuine transmitters of scientific knowledge, and that scientific texts clarify the history of their origins as well as the other way around. In the second place, he contributes to our understanding of what it means to claim that mathematics is not only a rigorous science but also a special form of discourse. The importance of this, it seems to me (though here I am certainly struggling with many obscurities in my own thinking), is that it is the job of mathematics teachers to establish the conditions for the successful induction of students into the mathematical discourse as much as it is to attempt them to lay down in their minds the fundamentals of the mathematical science. The two objectives are not in conflict and ideally complement each other, though traditionally only the second has been admitted. (p. 54)

Thus, I reconsider my view of how the two bridges need to meet. I intended to turn the mathematics into a story. Teachers were mathematizing the story through tasks; they were applying the mathematics. Perhaps these are not the only approaches. Wheeler reminds me that the gap between mathematics and story can be bridged through language. In the teachers' story, language and mathematics are implicitly related. But it is possible not to see the mathematics in the language. Whereas I am seeing the mathematics in the everyday word of *some,* and the teachers are perhaps seeing the mathematics in this word, the students most likely cannot yet see these relations.[24] Therefore, the story written by the teachers can promote discussion about mathematical discourse.

So how does the story created by the teachers align the narrative and paradigmatic worlds? Perhaps its appeal lies in the way it helps its reader and hearer engage in the story and wrestle with what the story means. This story involves significant philosophical problems arising throughout the history of mathematics. These are humanly contextualized issues: What are numbers? Why do we use numbers? How might they have been invented? What would a world be like without numbers? How do we engage in mathematics? What is mathematics? The story illuminates relations between *Thou* and *It* by provoking thought about mathematics as a humanity and as a science. Pondering the philosophical aspects of the story and engaging in paradigmatic tasks of solving related story problems or creating number equivalencies can develop robust relations among *I*, *Thou*, and *It*.

What, then, are the salient qualities of this story? As already described, the story encourages philosophical contemplation. It is compelling and the title contains a question but this is not posed as a problem within the text itself. . There is no expectation of getting a correct answer. Beginning long, long ago, the story is reminiscent of a folktale. Eco (1994) writes:

> If a text begins with "Once upon a time", it sends out a signal that immediately enables it to select its own model reader, who must be a child, or at least someone willing to accept something that goes beyond the commonsensical and reasonable" (p. 8).

Even as a parable, the tale falls short because "once upon a time" or "long, long ago" is reminiscent of "going beyond" normal expectations making it an inherently hyperbolic genre rather than a parabolic one.

While the story is about ordinary people doing common everyday activities, there is an anticipation of something that provokes surprise. This comes in the form of the content focus on language and communication. As a historical story, the tale falls short. There is, at best, a nod to a historical time and place but it does not specifically highlight an event or person. This transcendent quality of the story recognizes the context across different eras.

Referring to work by Rudolf Bultmann, Donahue (1988) distinguishes between folktales and parables by comparing narrative style, character portrayal, and plot devices. As opposed to folktales, parables are characterized by their concise narration, by their use of repetition to maintain the action, and by their lack of conclusion where certain issues are left unresolved. Many of this particular story's features, as identified by the teachers, are similar to these characteristics of parables. Their story includes narrative illustrations of philosophical ideas of mathematics encountered in everyday life. It is a teaching tale and gets its truth, as well as its potential to educate, not from an exposition of facts but from the open-ended and metaphorical way it provokes interpretation and contemplation of the relationship between people and mathematics.

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[24] In Chapter 6, I present Walkerdine's discussion of why the term *some* is mathematically significant for young children.

The metaphor *mathematics is story* appeared to break down when it was interpreted solely as narrative. As the metaphor *mathematics is history* was introduced, my intention was to add a paradigmatic element by blending two dimensions of mathematics: logical deduction and analogical truth. When this story was used as a teaching tale, the teachers mathematized it in a variety of ways that emphasized the paradigmatic mode. By creating and implementing this story, the teachers seemed to engage in both dimensions of thinking.

The teachers used language as a way into examining why numbers were needed. The philosophical discussions were seen by the teachers as necessary even when these discussions were not mandated as part of the official school curriculum. While the philosophical aspect of mathematics is not emphasized by any of the objectives contained in the provincial curriculum, it does represent an area of mathematics that requires further investigation. How is this a teaching story for the teachers? How did the creation of this story prompt professional growth?

Shifting Relations to *It*

My impetus for creating a mathematical story with the teachers was to encourage new relationships with *It*. Following the eighth group meeting, I interviewed each teacher individually in order to understand how they were now relating to *It*. One of the questions I asked was related to their perception of their own professional development in mathematics. Three of the teachers emphasized the philosophical nature of their learning about the history of mathematics. Martha told me that she had "never spent a lot of time thinking about how we arrived at where we are in math today" and had not considered questions about where mathematics came from. Although she found learning about the history of mathematics fascinating, she was puzzled and perhaps embarrassed by her lack of knowledge: "To be quite honest at times it made me feel really uneducated. How did I get here and didn't know that? Or how did I get here and I didn't ask that question? Why didn't I think of that?"

Similarly, Denise suggested her exposure to the history of mathematics within our study increased her interest in reading on the history of mathematics because "it makes you look really differently at it." She had never considered the possibility that other number systems existed:

> I actually had my eyes open to many things about numbers, number systems, where numbers come from, where zero developed. These were some things I hadn't really thought about because we've always just had our zero to nine and our base ten and away you go. I never even thought of why we developed roman numerals and why they were used the way they were, how people communicated back then.

Barb viewed the history of mathematics as another path to teaching mathematics. She introduced her students to Fibonacci and Newton, mathematicians she considered important. She highlighted the importance of learning about the philosophical dimensions of mathematics:

> [My participation in this study] will change the way I teach math. I will have more philosophical discussions when it comes to numeration. With children. And not just with those who I consider to be brighter kids, who I feel can go in that direction, but with the whole class.

Christine focused on the dynamic nature of her relationship with mathematics. Although I had brought in many resources on the history of mathematics throughout this project, she did not have a clear idea of what we were creating until the final stages. During the interview, she showed me a book "written about the people who came up with the math ideas. It's a biography of each person, [stories of] the people. I didn't even know that exists. It was just for kids." I found it interesting that this particular book was one that other teachers in the group had discussed and that I had brought in as an example of mathematical stories. It was not until she was enacting the lesson that she was able to think deeply about what teaching through historical stories in mathematics might mean.

Early in our project, Martha had commented, "The more things change, the more they stay the same." Many of our conversations investigated her claim. During the interview, I asked Christine for her response to this statement. She suggested, "I'm not sure if I would use that as my way of looking at my whole life" and then rejected Martha's claim because it did not describe her experiences of teaching in general and her experiences of learning mathematics. She did not believe that her new way of knowing mathematics would "reverse itself." While she "didn't expect it ever to change in the first place," she suggested that "the fact that it did is significant." Christine personalized our study of the history of mathematics by investigating the history of her relationship with *It*. She recognized that her own view of mathematics had changed and inferred that history informed the future but did not control it.

Shortly after this group meeting, Denise sent the following request by electronic mail:

> I am hoping to find out from you as soon as possible if these changes can be made to the story. [The illustrator] has been researching number and she found out a lot about ancient Egypt. She thinks we should try to make the story more historically correct by changing the names in the story to more Egyptian names of the time (she spoke to an Egyptian friend and has gotten some good ideas). The clothes etc. shouldn't be hard; she has picked up a couple of really good picture books from [the bookstore]. Also, there is no mention anywhere about potatoes way back then but there is lots of info about onions. So...should we change potatoes to onions, the characters' names? Let me know. She'd like to get started. Thanks.

The overwhelming response from the teachers was to proceed with adaptations to the story by having it take place in ancient Egypt. Consequently, the illustrator chose new names for the characters and incorporated details of Egyptian life from 3000 BC. For example, characters were named Neferet (beautiful woman), Majed (worthy of praise), Wosret (powerful one), and Nubi (gold worker). The houses in the illustrations were built of mud bricks, the characters wore white linen clothing, and the market sold barley, wheat, lentils, cucumbers, beans, leeks, onions, garlic, figs, and grapes (see Appendix F for the revised story).

When Denise presented the story drafts to the group, the changes were enthusiastically accepted. Moreover, the teachers were impressed with the quality of the story. I was intrigued by this since the teachers had firmly resisted my suggestions to align the story to a specific historical period and now they totally accepted the changes. Perhaps they recognized that the nature of the story had changed. The quality of a folktale had been replaced by the reference to a specific historical era. The story became more parabolic as these historical details served the purpose of situating the story in a believable context of living without numbers and directed the reader or hearer's attention to the teachers' intention of inspiring philosophical contemplation of mathematics' necessity.

While the content of the story pointed to the necessity of a formal number system, it also emphasized the human agency in the creation of this system. The teachers appeared to be considering mathematics as a humanity. How did this process of writing a mathematical story and identifying its features influence the teachers' views of mathematics? To what extent was this story a teaching tool for them? How did the creation of this parable stimulate a re-imaging of mathematics? I investigate these questions in Chapter 8.

Interlude 3 – Working on relations of *Thou-It* through attention to *I*

Teaching Without Children

> *Through the* Thou *a man becomes* I. *That which confronts [a person] comes and disappears, relational events condense, then are scattered, and in the change consciousness of the unchanging partner, of the* I, *grows clear, and each time stronger. To be sure, it is still seen caught in the web of the relation with the* Thou, *as the increasingly distinguishable feature of that which reaches out to and yet is not the* Thou. *But it continually breaks through with more power, till a time comes when it bursts its bonds, and the* I *confronts itself for a moment, separated as though it were a* Thou*; as quickly to take possession of itself and from then on to enter into relations in consciousness of itself.*
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> *(Buber, 1923/1958, pp. 39-40)*

Buber writes, "Through the *Thou* a [person] becomes *I*." Throughout much of this research, I focused on relations of *Thou-It*. However, my *I* evolved tremendously because of my interactions with *Thou*. Two relational events that profoundly impacted me took place on December 2, the day of our last group meeting. The first episode involved my realization that I was no longer a teacher. The second incident prompted me to reconsider my role in relation to our learning community of teachers. While this interlude was difficult to write, I include it because it describes my *I* becoming.

During this project, I experienced collisions between my identities as teacher, participant, teacher educator, and researcher. From the beginning of my work in the classrooms, it was apparent that the teachers were not viewing me as a teacher. They introduced me to their students as a guest, as a professor from the university, and as a researcher of mathematics. I introduced myself to the students as a former teacher, making it clear to them, however, that I was not a substitute or supply teacher. However, during the project, I remained a classroom guest and daily schedules written on the board announced me as such. I brought with me special stories and tasks for the students but relied on the classroom teacher for assistance with the classroom routines. Each week I collected student work for my research but was not compelled to provide feedback to the students in the form of grades or comments. During the project, my separation from the teachers became more evident and I became conscious that I was not a teacher.

Tensions arising from my ambiguous role in the classroom evoked a strong sense of discomfort for me. I entered the project as a teacher at ease in the classroom but soon realized that I was also a researcher. While student learning remained my priority in the classroom, I found myself observing the students' behaviour and listening to their conversations without intervening as much as I would have done if these were my own students. My interests in the classroom were not focused on the attainment of curricular objectives. Rather, I was gathering data that informed my sense of *It*.

Both the tension I experienced during my work in the classrooms and the teachers' views of me influenced my sense of self. Like Buber suggests, my *I* was confronting itself and my consciousness of *I* was growing clearer and stronger.

December 2 was the last day that I taught in the classrooms of these teachers. As I completed my final lesson and walked to the parking lot, I was overwhelmed by a sense of sadness and grief. I sat in my car for several moments, tears streaming down my face. I knew that I was never going to have my own students again. I was no longer a classroom teacher and knew I would not return.

At the beginning of my endeavour to provide release time, I was a teacher. However, as time progressed, I felt disconnected. I did not know the students. The students were hesitant to participate in the group discussions and did not seem ready to work in small groups. I spent much of my time motivating students to engage in the mathematical tasks and to complete the assignments. I made seating charts so I could learn their names. I tried to interact with them but the time was too short. Gradually throughout the months, I developed a tentative rapport with many of the students. But it took hard work and my previous taken-for-granted relationship with my former students was lost. I had been a teacher and was painfully aware that what I was doing was not teaching. I felt like a visitor. At the end of my time with the students, I believed that I still did not know them.

On December 2, I cried because of the *I* that I was leaving behind. I grieved the loss of me as a teacher. I knew I was terminally childless.

Buber writes, "Relation is reciprocity. My You acts on me as I act on it. Our students teach us, our works form us" (p. 67). Even though I did not encounter the same deep relationships with these students as with my former students, the students I was visiting taught me that I could still relate to them. During the final interview with Christine, I shared my angst with her. She expressed surprise and commented:

> It's not often that kids have someone that comes in to do something special with them. And that's how my guys looked at it. This was a special opportunity. This was something that not everybody gets to do. […] The kids did not feel that they were not known by you. That was just a gift you brought to them."

Through Christine and her students, I began to see myself differently. I was no longer a teacher but perhaps I could still enter into *I-Thou* relationships with the students.

Following this lesson on December 2, I drove to Martha's school for what was intended to be our final group meeting. The mood within the room was cheerful and festive as we snacked from delicatessen platters of food. As the teachers shared with each other their impressions of the story implementation, they were animated and highly engaged. For nearly an hour and a half, the teachers talked. When the conversation lapsed, I mentioned that "the other thing I need to do is have exit interviews with each of you individually like I did in spring." Not one person spoke for a number of minutes. Fearing that they were unwilling to be interviewed, I remained desperately quiet and waited for someone to break the silence. Finally Barb asked quietly, "What if we don't want to exit?" They laughed and I joined in because I was completely relieved. We agreed to hold "interim" interviews and to continue our meetings once a month until the end of June.

This group meeting was a pivotal moment for me. I realized that relationships within the group had developed into deep friendships and *I-Thou* relations were robust. These teachers were willing to continue our study of the history of mathematics despite my choice to discontinue the provision of release time for them[25]. As I began to reframe my identity as a teacher educator, I felt they had given me a gift. For the remainder of the school year, I had an opportunity to grow into this new role within the safe context of our ongoing relationships. Out of my sadness of losing my teaching self, these teachers offered hope to me. Moreover, without the collective *Thou* in this study, there would have been no *I* becoming in such a personally significant way.

Themes of Relations

Martin Buber writes extensively about relations, emphasizing the importance of dialogical encounters. His philosophical ideas seem directly connected to his life experiences. As editor of *The way of response* (Buber, 1966), Glatzer suggests that Buber's focus on relational responses was "born out of crises in his own life" and identifies one of these as "the crisis of the young boy—son of divorced parents—told that the mother would never return to him" (p. 10). Similarly, Friedman (1991), the translator of most of Buber's books and personal friend of his, links Buber's preoccupation with relations to the loss of his mother as a child. According to Friedman, this crisis was a decisive moment in Buber's life and prompted him to coin the term *Vergegnung* to indicate a mismeeting or a failure of a real meeting between persons. Buber's contribution to my own understanding of relationships is apparent. Knowing something of his history helps me as I grapple with my own yearning for community.

The two most significant communities in my childhood were my school and my family. However, I experienced childhood isolation because of a lack of sustained participation in a school community. The transient nature of my family provided momentary glimpses of dialogical encounters and authentic *I-Thou* relations as I moved temporarily in and out of relationships. My first encounters with mutual meetings occurred when I began my teaching career. The community of teachers was very important to me as I was becoming a teacher. My childhood experiences of difference were replaced by adulthood encounters of acceptance. I entered teaching with my whole being and my students became my children. Certainly when I transferred from a junior high school to an elementary school, my nurturing disposition was intensified.

What I recognize now is that there was a sense of distance among my school relations. My "children" were temporary, soon to become the children of someone else when the school year ended. During the year, I could leave my students at the end of the day and suspend my participation in the relationship. I had my own "children", yet I did not have to commit to a lifelong relationship. My relationships with the students were yearlong *I-Thou* commitments, sustained for a short period of time until they ended in June. I could stay involved in this community but remain distant at the same time.

My childhood relationships within my family cannot be characterized by Buber's notion of *I-Thou* relations. While I had a strong relationship with my two younger sisters, the relationship with and between my parents was one of mismeeting. Growing up, I decided that I was not going to get married and I was not going to have children. While I have entered an *I-Thou* relationship within marriage, I have chosen to remain childless. As a teacher I have been conflicted because of this conscious decision. There is great dissonance between my desire for *I-Thou* relations within a school setting and my obvious rejection of these relations in my personal life.

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[25] They respected my need to write this dissertation.

Failure to encounter a relationship with his mother may have motivated Buber to seek out dialogical meetings with others. He writes about the innate instinct to make contact and describes this first contact as meeting. More significantly, he identifies the child's soul as longing for the *Thou*:

> In the beginning is the relation—as category of being, readiness, grasping form, mould for the soul; it is the *a priori* of relation, *the inborn Thou*. The inborn *Thou* is realised in the lived relations with that which meets it. [...] The development of the soul in the child is inextricably bound up with that of the longing for the *Thou*, with the satisfaction and the disappointment of this longing, with the game of his experiments and the tragic seriousness of his perplexity. (pp. 38-39)

Buber's themes of encounter, dialogue, relations of *I-Thou,* and relations of *I-It* are his responses to "the tragic seriousness of his perplexity." His mismeeting with his mother and the absence of this primary relation in his life did not limit his acceptance of others. Indeed, his relations with both his grandmother and his wife taught him the possibilities of dialogical interaction. Perhaps his experience of the lack of this relationship with his mother helped him to become more attuned to otherness and relations. Certainly, his living and meeting was evident to many who encountered him. Glatzer writes:

> A biographer [of Buber] would have to recognize the great loneliness in Buber's life, and value in his work his valiant defiance of pessimism, and his affirmation of a faith in God that commanded him to be faithful to man—to the last. (p. 13)

Like Buber, I have encountered *I-Thou* relations with my aunt and with my husband of twenty years.

My aunt, my mother's sister, was extremely independent yet loved spending time with our family. Indeed, when she moved to the city in which we were living she became more involved in our family. Since I have moved away to pursue my doctoral degree, she has had a series of small strokes and has been diagnosed with a degenerative brain disease. In spite of having difficulty speaking and walking, she has remained a gracious woman. Although she has experienced a loss of freedom and independence, she seems able to accept her situation.

I remember my aunt's care for me when our family moved during my eighth grade of school. While my younger sisters found companions in the neighbourhood, I had no friends. I was at an age where I imagined that my mother did not, or could not, understand me. My aunt made many trips to visit me. On each visit, she brought me a gift. I remember receiving a t-shirt with the slogan "Everybody needs a friend" emblazoned on it. She gave me a poster reading, "Just when you thought it couldn't get worse, it does." She brought me a ceramic holder that had my name painted on it. Her quirky sense of humour helped me gain perspective and confidence. Her compassion and strong sense of family taught me to balance independence and community. Her work as a nurse showed me how a woman could nurture others professionally.

In many ways, I am like her. We share a similar sense of humour, a commitment to independence and community, and dyed red hair. Oakley (1984) writes that breaking "out of the routine treadmill of female existence" depends on identifying with an adult model who sets "a powerful example of involvement in the nondomestic world" (p. 12). For me, this adult model was my aunt. However, unlike my aunt, I chose to marry. I enjoy both independence and companionship. I am grateful to my mother for encouraging this relationship between my aunt and myself and I am grateful to my aunt for this encounter.

Through my encounters with my aunt and my husband, I have become less hesitant to share my vulnerabilities. Buber writes:

> Every actual relationship to another being in the world is exclusive. Its You is freed and steps forth to confront us in its uniqueness. It fills the firmament—not as if there were nothing else, but everything else lives in *its* light. As long as the presence of the relationship endures, this world-wideness cannot be infringed. (pp. 126-127)

The tensions of my childhood relationships, the tensions between my need to "mother" and my choice not to have my own children, and the tensions between my desire for a home and my life experiences of impermanency continue to confront me. However, the challenges of relationships within these settings have helped me to "enter into relations in consciousness of itself" as Buber suggests. In this study, my *I* was realised through the intense work on *Thou-It* and in the "lived relations with that which meets it." My transition from teacher to a teacher educator continues to be a profound experience of evolving identity. I am now more confident of my role as a teacher educator and researcher within the classroom and can envision a way of living in relation to both teachers and students within this setting. Through the *Thou,* I am becoming. Perhaps I can enter the classroom as a favourite aunt, offering new possibilities for *I-Thou*, *Thou-It*, and *I-It* relations.

Chapter 8 – Re-imaging Mathematics

Mathematics is...

> *Barb: I didn't know a lot that math began with the Egyptians. I really thought that it was Greek and during the Greek and Roman times. So I guess maybe I'm thinking more about Roman numerals. So certainly that part, the history of math, has changed. As far as the nature of math being important to everyday life and the way that we make sure that children realize how important it is to everyday life, I don't think that aspect has changed for me. But having children have a good understanding of where numbers came from or why we use them has changed. I certainly now see that that might be an important aspect, especially for those children who are afraid of all those algorithms that we set up at the beginning. [...] They see it more as something that's between people instead of just numbers on a page.*
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> *(Interview, January 25)*

In Chapter 1, I introduced my thesis statement concerning elementary teachers' views of mathematics. Considering mathematics as a humanity, I claimed that teachers' mathematical experiences and images of the nature of mathematics could be altered through immersion in the uncertainty and indeterminacy of historical stories in mathematics. By analyzing the relationships among *I*, *Thou*, and *It* within a particular professional development setting, I attempted to describe possibilities for teacher learning in Chapters 2 and 3. Initial metaphors offered by the teachers were investigated and the emerging metaphors of *mathematics is language* and *mathematics is story* that corresponded with recurring themes of our conversations were examined in Chapters 4 through 7.

My intent in this dissertation was to investigate three research questions. (1) What were teachers' encounters with mathematics when creating and implementing a series of lessons based on a historical story for elementary mathematics classrooms? (2) What were my experiences of working on the mathematics as a teacher educator? (3) How did we, as members of a learning community, experience our relationships with one another? Much of my writing in Chapters 4 through 7 focused on the first research question. The second research question was addressed in the Interludes and the third question will be attended to in Chapter 9.

In this chapter I present what I consider to be the most important conclusions of my study in relation to the first research question. My main claim is that participation in this project fostered alternative images of mathematics as teachers grappled with philosophical tensions in mathematics.

While my thesis seemed to be supported by the study, it became clear to me that the teachers were rooted in narrative ways of knowing mathematics and were resisting a paradigmatic orientation. I began to consider the limitations of the metaphor *mathematics is story*. Since we focused on mathematics as a humanity, there appeared to be little increased understanding of the paradigmatic dimensions of mathematics and mathematical ways of knowing. This leads to my second conclusion: Engagement with the history of mathematics by itself was insufficient to move these teachers toward the paradigmatic dimensions of mathematics.

To what extent did the creation of a mathematical parable (see Chapter 7) stimulate a re-imaging of mathematics and what shifts, if any, occurred in the teachers' views of mathematics? To what extent did study of the history of mathematics encourage a consideration of both paradigmatic and narrative ways of coming to know mathematics? I begin by investigating the mathematical images that resonated with the teachers.

New Metaphoric Images

In order to investigate teacher learning, I interviewed each teacher during the final stage of the project[26]. During the individual interviews that followed the eighth group meeting, two teachers offered a glimpse of how their participation in this project had impacted them. In the excerpt presented at the beginning of this chapter, Barb focused on the importance of mathematics to everyday life, additionally commenting that words were important and that mathematics could be viewed as communication between people rather than as numbers in isolation. Christine suggested that the experience "changed my hesitancy to participate in math-oriented conversations or work" and emphasized the enjoyment and fun she had experienced after a "life-long hatred of mathematics." These teachers were beginning to re-image mathematics.

Both Barb and Denise mentioned the significance of the parable that had been created. Barb commented, "We have something very valuable that will get other teachers thinking and talking about math in a different way." Barb valued the tacit philosophical content of the parable and felt that it would foster conversations about mathematics among teachers and encourage teachers to think about mathematics in a different way. She was impressed by how the story prompted consideration of the origin of numbers. Although she had a strong background in mathematical concepts, she had not encountered their historical underpinnings. She was familiar with symbols and counting devices used by other cultures such as Roman numerals and Chinese knots, but "even in all my years of teaching, there are very few stories in the textbooks about the origin of math."

Denise also emphasized the value of teaching the history of mathematics. The experience of learning about the history of numbers as an adult was significant to her because "now [the mathematics] makes so much more sense." Contrasting the inclusion of history in social studies and science classes with its omission in mathematics classes, she suggested:

> Going back to the roots and understanding where it came from would really give the kids a basis for their learning. Lots of other times I think they say, "Why?" [...] I think the history is really important in math, to even understand who developed these numbers and why they form them the way they do.

Again, the philosophical questions surrounding the origin of numbers were identified by Denise as important.

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[26] While the teachers in my study continued to meet with me for five months after this time (see Interlude 3), for purposes of my dissertation research, I decided to end my data collection at this point. Thus, I consider the final stage of the project to be the individual interviews that followed the eighth group meeting.

Barb and Denise referred to their story in relation to philosophical and historical dimensions of mathematics that they had not previously considered. Although Barb and Denise attributed their learning to the creation of the parable, later in the individual interviews, both emphasized the importance of the group conversations and my work with the teachers. The creation of a mathematical parable in conjunction with our group conversations seemed to prompt a re-imaging of mathematics.

During the interviews, I asked each teacher to express metaphorically how she viewed mathematics. I chose this question because I believed that a change in metaphor might indicate a change in perspective of the nature of mathematics. According to Lakoff and Núñez (2000), shifts in meaning can occur as new metaphoric connections are formed. I had invited each teacher to reflect on metaphors of mathematics at the beginning of the project (see Chapter 4) and I was interested in possible shifts in images of mathematics. Zwicky (2003) suggests, "In metaphor we experience a gestalt shift from one distinct intellectual and emotional complex to another 'in an instant of time.' A metaphor, then, is also a meta-image. It is multiply resonant" (p. 4 left). My interpretation of the teachers' responses reveals that while some metaphoric shifts were subtle, others seemed quite explicit.

Mount Robson[27]

At first glance, it looked as if Martha had not been influenced by our conversations. In the interview, I prompted her to consider her view of mathematics and she reiterated her metaphor of seeing mathematics as a mountain (see Chapter 4):

> I think it's still Mount Everest but I think I have more tools to be able to climb it. It's maybe Mount Everest that I've conquered once so I know I can do it again. Maybe it's not quite Mount Everest. Maybe it's more like Mount Robson or something. Not quite so big and cold and scary but, not for me as much but for a lot of kids, it still is a big, scary, cold mountain.

When I asked her how this metaphor was evident in her teaching, she suggested that her own conquering of mathematics provided an example of success to her students. Teaching mathematics involved "breaking it down for them so it's in smaller pieces, little bumps instead of big mountains."

I experienced many tensions when I listened to each teacher describe her view of mathematics. In this particular instance, it seemed the teacher-presented metaphor was stable and ingrained: Martha's description of mathematics appeared to be set in stone. Mathematics, like a mountain, could be conquered and could be broken down into smaller steps. In this interview, she offered the same metaphor for mathematics and her *It* was still viewed as a struggle. Her description did not appear to reflect our conversations about *mathematics is language* or *mathematics is story*. Although she had experienced different views of mathematics, she seemed unwilling to reconsider her own notions. She suggested that "the more things change, the more they stay the same" and described how she was imaging mathematics:

> Well, I think math is still a struggle for so many of these kids, the same way it was for me many decades ago. And this curriculum's changed, and the teaching

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[27] At an elevation of 3954 metres, Mount Robson is the highest peak of the Canadian Rocky Mountains. Mount Everest is the highest mountain in the world at an elevation of 8850 metres.

> tools have changed, and the resources have changed, the way we view math has changed, the way we understand thinking and brains and development and everything has changed, and yet these kids still struggle with the same things as I did.

For Martha, her experience of struggle remained a consistent theme. Even though she recognized that approaches to teaching mathematics had changed, the nature of mathematics as an *It* remained constant.

On closer inspection, what seems evident in her interview responses is a slight shift away from absolutism. Martha chose a mountain metaphor that was part of her familiar landscape and consequently might have been more accessible to her. She viewed her teaching role as providing guidance, equipping her students with appropriate tools, and encouraging them to make some assumptions. Giving students permission to "get the wrong answer" and acknowledging that "there's no answer that works for everybody" suggests that Martha was at least considering new ways for negotiating this mountain, even if, in essence, it remained a mountain. She said, "All I can do is show them there's a way over it. And that if you don't have the tool that you need to get over it, where to go to find that tool."

A Pattern

Like Martha, Barb did not radically change her metaphor of mathematics. Although she described her metaphor as a pattern, there were remnants of the bridge metaphor in her response:

> It's still a lot of patterning. And I guess I don't know if I could change that metaphor. The patterning helps between the bridge between the algorithm and what you actually do. That's what I mean by the bridging, is the bridging between the numbers and sociological aspects of problem solving.

Later in our interview, she explicated the relationship between patterning and bridges:

> So I'm still more and more convinced that patterning is really an important thing for children to look for in math, that it helps them. It's one of those bridges or connections that really helps them, both in the algorithm and maybe even in the stories, too.

When initially describing mathematics as a bridge, she suggested that the bridge did not end but continued in many paths. This idea was intertwined with what she was learning about the history of mathematics: "I'm thinking more of a road as a never-ending road that still continues to take you, and a road with many paths, of course. Because, for me now…the history of math is another path." Barb described this new path as a new teaching strategy and seemed to view the history of mathematics as a tool: "You have to have this pocket that's full of five or six different ways of looking at one concept." The history of mathematics was seen as one possible way of bridging mathematics and students' own experiences.

Another tension for me was that some of the metaphors were mixed and not fully formulated. For example, Barb's metaphor of a bridge was combined with that of patterns. Also, some of the descriptions given by the teachers in this interview appeared tentative as images of mathematics were presented as similes rather than metaphors. This is evident in Martha's suggestion that "It's still Mount Everest" but "maybe it's more *like* Mount Robson." Similar to Pesman (1991) who offers the notion that "a mixed metaphor is an infidelity" (p. 216) because a promise of union is broken, my discomfort arises from the implied dissonance between two concepts placed together in mixed metaphors. I tend to seek coherence and often find mixed metaphors confusing because they violate a tradition of conscious rhetoric where coherence is prized. However, Pesman challenges the "idealized, commonsense notion of reality" (p. 224) by claiming that this mixing may represent emerging thought:

> Perhaps consciousness *cannot* leave room for "genuine mysteries." What is called mixed metaphor [...] is the coming into consciousness of a mixing that goes on all the time, a consciousness that offends our sensibilities because it "calls attention to the device" and perhaps might reveal the inexplicable basis of our worldview. (p. 243, italics in original)

Calling "attention to the device" encourages a poetic consideration of the figurative language that is being encountered rather than an identification of the constant relationship that "should ideally be maintained between the metaphorical domain and that of which it is predicated" (p. 218). Mixed metaphors present a bricolage of similarities arising out of an attempt to make sense of new ideas and situations. In this way, shifts in metaphors portrayed by this mixing may point to a grappling with a "genuine mystery" before it is articulated or consciously developed into a coherent theory of reality. Thus, evidence of mixed metaphors may point to shifts in the way teachers were viewing mathematics. It is this mixing of metaphors that is most apparent in the descriptions of mathematics as a puzzle offered by Denise and Christine.

A Puzzle

In this final interview, two teachers described mathematics as a puzzle. At the beginning of our conversation, Denise offered the metaphoric claim *mathematics is everywhere*. Throughout this study, she claimed that she was able to see math in everyday activities and had used a range of literature in her mathematics classroom. Certainly, as the author of their story, she viewed the terms *some*, *many*, and *a few* paradigmatically. However, she shifted tentatively to consider mathematics as a puzzle and described this as a simile:

> Math is like a puzzle, I guess, that we have to try to solve. With anything that we do whether to make curtains or buy a car, or anything, it involves math. So I'd say math is like a puzzle. [...] You have all these little pieces and they have to go together to get a solid grasp of a concept. So a whole bunch of life experiences and chunking of information, paperwork, and lessons, and everything that brings your thoughts together to complete the puzzle.

Probably more evident in her teaching was the idea that finding missing pieces of the puzzle helped her students make mathematical connections. This was reminiscent of the bridge metaphor where connections to the mathematics were emphasized:

> I'm seeing connections being made all the time with the kids. That they might have a couple of pieces of the puzzle and they might be missing one thing. And then all of a sudden you hear, "Oh, now I get it."

For Christine, mathematics was a puzzle of patterns. She blended her metaphor with our conversations about patterning. She suggested that a puzzle "invited you to discover something and put pieces together for a purpose" and emphasized the "fun element." By proposing that puzzles have patterns, she opened up possibilities for viewing mathematics as something that was more than functional: "It is functional, of course, but it's not my whole way of looking at it any longer."

Her metaphor shifted when she combined descriptions of puzzles that involve putting pieces together and puzzles found in a book. She portrayed puzzles as mysteries and suggested that the process of solving puzzles involved "patterned ways of thinking," persistence, and strategy. Another shift occurred when she realized that her metaphor broke down:

> I don't think of puzzle building as something where you communicate but you can't make connections very well in math without the terminology meaning something or the wording meaning something. So the better that my students understand the word that I'm using, the better they know what they're doing. And the less discomfort there is when you're going about a task. So in some way that wording is a strategy element, too.

When describing mathematics as a puzzle, both teachers offered unformulated metaphors. In particular, Christine recognized the limitations of her metaphor and added a description of mathematics as involving communication. She emphasized patterning and connecting. Our group conversations about language and communication seemed to influence her view of mathematics.

When Burton (1999) asked professional mathematicians to describe how they came to know mathematics, the metaphors of the jigsaw puzzle and geography (in many cases a mountain) were predominant. Like the mathematicians in Burton's study, it seemed that many of the teachers were thinking about mathematics in ways consistent with the views held by people adept at doing mathematics. The mixed metaphors Barb, Denise, and Christine offered pointed to shifts in their images of mathematics.

Clearly, teachers' images of mathematics were impacted by the conversations taking place during the creation and implementation of the parable. Indeed, the metaphor *mathematics is story* prompted a consideration of the humanistic dimensions of mathematics and contributed to a varied re-imaging of mathematics. Our work together as a group provided an opportunity to investigate philosophical questions of "Why?" that are singularly absent in paradigmatic ways of knowing mathematics. For these teachers, the philosophy of mathematics provided a bridge to mathematics.

However, I was unclear about the role of the history of mathematics in providing a narrative route into mathematics. The teachers' resistance to incorporating historical perspectives that focused on the paradigmatic nature of mathematics within their story was described in Chapter 7. By classifying their story as a parable, they addressed its philosophical focus and its pedagogic intent but failed to see the need to relinquish the hyperbolic quality of their story as a folktale.

When I introduced this chapter, I claimed that the history of mathematics proved insufficient to help move this group of elementary teachers toward the paradigmatic dimensions of mathematics. The narrative mode of thinking was accessible and comfortable for these elementary teachers. However, their experiences with paradigmatic dimensions of the history of mathematics were limited. In what ways, then, does the metaphor *mathematics is story* fall short?

Mathematics is Story: Limitations of the Metaphor

For me, the metaphor *mathematics is story* emphasized historical stories of mathematics (see Interlude 1). During the closing interviews, I asked each teacher how she viewed her growth in the area of the history of mathematics. The responses were general and vague. It appeared that the teachers were limited in their understanding of the history of mathematics. Christine said that although she had a general impression of historical ideas, she was "still learning" and had "a long way to go." She identified learning about early number systems and found the different bases "really strange." This seemed to prompt a paradigmatic way of knowing mathematically as she focused on learning "how it worked."

> I think that's one of the historical things that's really maybe just provoked me to want to think a little bit more about how I can share that with [my students]. But I don't think I've gained a huge amount, because I don't remember as much of the reading as I would have liked. So I need to go back though and read some of the things more to absorb them.

Christine was motivated to find out more about the history of mathematics but our study of the history of mathematics was inadequate in providing sufficient opportunities for her to grow in her paradigmatic understanding of mathematics. This project became an introduction that stimulated her interest and provided general knowledge but failed to enhance her view of the paradigmatic nature of mathematics. When asked if the history of mathematics was necessary, she responded:

> Not necessarily. It's one way to frame it in a story. I find it easier to frame some of the math in historical story than I do just in a story. That would be how I would look at it. So for me it would be easier.

Likewise, Martha found the conversations about the history of mathematics stimulating:

> I never spent a lot of time thinking about how we arrived at where we are in math today. I guess because I've never been asked the question, "Where did it come from?" I guess if I ever sat down and thought about it before, I wouldn't have had an answer. Now, I'd be able to fumble through some kind of an answer or at least know where to look.

Yet, her familiarity with the history of mathematics was limited and she would have to "fumble through some kind of answer" to the philosophical questions we posed in the group. When asked what areas in the history of mathematics she would like to study further, she replied, "I don't know what I don't know."

Much of our initial group conversations focused on the origins of numbers in earlier times. In the final interview, Christine remarked:

> It's been good to get a sense of how much of Eastern knowledge we've ignored or downplayed or not given credit to over the years just because we're western in our thinking. I'm not so sure that's a good thing.

However, her understanding of the developments of number concepts prior to the era of ancient Greece and Rome was limited. Martha was preoccupied with the Greek and Roman civilizations and the notion that mathematical developments were not universally known or accepted between these societies. She believed mathematics began with the Greeks and Romans. Barb suggested that her understanding of the history of mathematics had changed in a small way, again emphasizing the importance of Greek and Roman eras:

> I guess not much has changed other than the fact that I didn't know a lot that math began with the Egyptians. I really thought that it was Greek. And during the Greek and Roman times. So I guess maybe I'm thinking more about Roman numerals. So certainly that part, the history of math, has changed.

At first I thought that Western histories of number development were being emphasized because teachers were making connections with their social studies curriculum. However, none of the teachers in this study were teaching curriculum units on Greece or Rome. Their familiarity with Western ideas leads me to conjecture that these Western notions of mathematical thinking were deeply ingrained. While Denise seemed to recall our conversations about tally marks and evidence of counting in prehistoric times, she commented on her understanding of Roman numerals and philosophical questions arising from our conversations:

> I never even thought of why we developed Roman numerals, and why they were used the way they were, how people communicated back then using just like we said with the some and some more and things like that. So it's actually made me more interested in reading on the history of math. I didn't know that I'd ever be interested in it. But I am now because of the exposure that I've had to it. It makes you look really differently at it. And the kids are interested in those old stories, how people used to just count with tally marks and things like that

The choice to portray the development of numbers in their parable as Egyptian represented an inaccurate understanding of the history of numbers, even given what we had already explored together. Denise admitted that she did not know where and why the symbols had come into being and wanted to know who had created them. Even though she was the author of the story, she recognized that her depiction of the setting in Egypt was fictional. She desired more specific knowledge about the history of mathematics.

By presenting the metaphor *mathematics is story*, I had expected to find their strength in language arts to carry over into mathematics. I thought that their ease with the story might facilitate a comfortable way in to working with the mathematics. This failed to happen in the manner I had anticipated. The story was not mathematized; it did not include the mathematics to work on. There was no mathematical problem. During the implementation phase, the teachers saw many possibilities but sometimes these were not in mathematics but rather in language arts. In this setting, through this process of creating a parable, mathematics appeared fragile. It seemed clear to me that if mathematics were not attended to, it most likely would be overlooked.

It appeared that many of the teachers in my study were unable to see mathematical possibilities when implementing the parable. A notable exception was Barb. She had a stronger background in mathematics and introduced her children to mathematicians like Fibonacci and Newton. She emphasized that many of her students "hadn't even thought about mathematicians as people who have invented something to describe something." She was able to expand the ideas that emerged within our group conversations and recognized the paradigmatic strengths of these ideas. When asked how she had worked with the history of mathematics this year with her students, she commented that she had applied patterned way of thinking when they were talking about base-10 systems. She proposed that patterning was "one of those bridges or connections that really helps them. Both in the algorithm and maybe even in the stories, too."

Whereas Barb was able to engage in the paradigmatic dimension of mathematics while studying philosophical ideas of mathematics, Martha was convinced that her facility with the basics of mathematics eased her ability to engage in philosophical and historical conversations:

> Maybe I'm finally reaching a comfort level with math that I can get into more of the philosophical historical stuff whereas up until now I couldn't go there because I was still struggling with the basics of it. I don't know.

An interesting point about her comment is that she felt her content knowledge enabled her to participate in philosophical and historical investigations in mathematics. Yet, she was one of the teachers who struggled with mathematizing the parable when implementing it in her classroom and the metaphors she used to describe mathematics focused on her struggle with mathematics. Perhaps it was actually her comfort level with story rather than with mathematics that allowed her to participate in these conversations.

During the interviews, two of the teachers seemed to be able to articulate the paradigmatic dimensions of mathematics. Barb interpreted the metaphor *mathematics is story* by linking problem solving, algorithms, and story:

> To me, math is the story of problem solving. And for children and even for ourselves, we have to connect it to every day. So because I consider stories as things we do every day, then math is always a story. So, how we do it, how do we use math, every day, it's a story. Whether you're measuring something, whether you're going to buy something, whether you're using it to solve a problem of area or perimeter, just simply sharing money, putting things in groups, how much do I have left, how much do I have all together? All of it is story. So to me the problem solving is just the essential part of it. That's to me what math is all about. The algorithm just simply describes what it is that we're doing in our story.

Denise suggested that students would benefit paradigmatically from working with the parable and would improve their relationships to *It*:

> I think it would make their knowledge base stronger. It's kind of like the roots of a plant. You know if you have all of that, you'll have a stronger plant. I think you'd have stronger math skills. You just see kids look inside now. They're just so interested.

Denise connected their story to the paradigmatic dimensions of mathematics. She expected the story to help students develop stronger mathematics skills and knowledge. When describing mathematics as being culturally dynamic, she recognized that children in other parts of the world had different number systems and symbols. Yet, she alluded to the universality of mathematics by suggesting that the processes of mathematics remained constant. Her recognition of the systematic nature of mathematics was evident: "You're always going to get the same answer doing the same process."

However, shifts in her thinking about mathematics as solely a systematic process were evident in her description of how the knowledge base became stronger. She offered a tentative simile: "It's *kind of like* the roots of a plant." Perhaps she was trying out a new image of *mathematics is organic*. Her relationship to *It* seemed to be altering.

For Barb, a strong background in mathematics helped her make sense of the metaphor *mathematics is story*. Denise's paradigmatic way of thinking was prompted by her investment in writing the story. However, both teachers commented on how strongly the group conversations had fostered these paradigmatic turns. Thus, *Thou-It* relations seemed strongly connected to *I-Thou* relations.

In this chapter, I have focused on *Thou-It* relations as I claim that teacher-held images of mathematics changed through the process of studying the history of mathematics, even though this focus on the history by itself was not an adequate stimulus of change. What else appeared necessary for prompting teacher learning? How did relations among the teachers and myself encourage professional development?

Although narrative ways of knowing mathematics were embraced by the teachers, relations of *I-Thou* were integral to this work. I moved away from the strong focus on the history of mathematics evident in Chapter 1 because of resistance by the teachers. Tensions emerged because I was rooted in the paradigmatic and the teachers were rooted in the narrative. I had to go further into the narrative to make the connections than I was happy with but strong *I-Thou* relationships enabled me to offer paths for crossing the divide, thereby increasing possibilities with respect to the paradigmatic experiences of *It*.

I claim that teacher learning occurred in this professional development setting because relations among *I*, *Thou*, and *It* were robust. This, the third conclusion of my research, I discuss in Chapter 9.

Chapter 9 – Expanding Relations among *I*, *Thou*, and *It*

The *It* of Professional Development

> *Barb: I was part of this group and it was a great, positive experience.*
>
> *Gladys: Talk a little bit about what made it*
>
> *Barb: Positive? I think, number one, the diversity of the group. Number two, the fact that we were all open to what each other had to say. We were respectful to each other. No one ever felt that they were perhaps better than the other. We all came with a very open attitude. We all shared very well. No one said, "I do this in my classroom so I don't want you to do it in yours." We had that wonderful collaborative attitude and mutual respect. Not just for the difficulties in the classroom but the whole process of how to become a better math teacher.*
>
> *(Interview, January 25)*

One of the dimensions of this study included the *It* of the professional development setting. In Chapter 2, I described the way that I envisioned nested relations within this research:

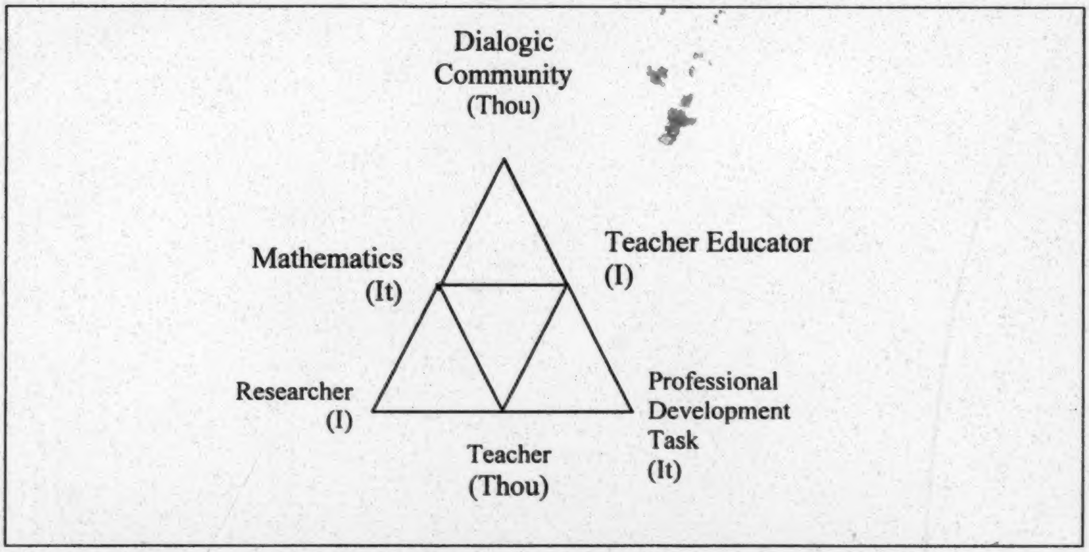

Figure 9-6. Nested relations within professional development settings

The inner triangle represents relations explicated Chapters 4 – 8. Points where this triangle touches the outer triangle represent themes described in each of the interludes. In this chapter, I consider the outer triangle consisting of the dialogic community, the researcher, and the professional development task of creating and implementing a series of lessons based on a historical story. Here the *It* becomes the learning task that includes the *It* of mathematics. My focus in this chapter is on the *It* of the professional development setting.

One of the most significant aspects of this research study for me was the quality of the interaction among participants. Drawing on Buber's (1923/1970) philosophical ideas, I suggest that there were authentic meetings among us at times and that we participated in mutual relationships. Each of us acted on the other in an adaptive process while encountering professional relations of *I-Thou*.

In the opening excerpt of this chapter, Barb identified the collaborative and respectful nature of our interactions as part of her positive learning experience. The focus on becoming better mathematics teachers was highlighted. The quality of these *I-Thou* relationships while working on *It* contributed to our professional learning. Therefore, a third conclusion of my study is that teacher learning occurred in this professional development setting because relations among *I*, *Thou*, and *It* were robust[28].

During the final interviews, I asked each teacher what made the most impression on her as a learner. Barb's response captures those aspects of the study that all of the teachers mentioned in their interviews:

> I just think the collegiality of the group. That, for sure, has probably had the most impact. The discussions. Certainly you bringing the idea of the history, just introducing that to the children. And then all of the discussions that we've had together as a group have really got me looking at math in a different way. [...] That collegiality has been really important. I'm thinking about the visits to the classroom.

Like Barb, all of these teachers identified the relationships they had formed through my participation as a teacher in their classrooms, the group conversations, and the classroom visits as being important dimensions of their professional development. The form of the experience was as significant to them as the content of their study. Indeed the intertwining among relations of *I*, *Thou*, and *It* was evident in their responses. In what ways was this encounter of professional learning fruitful for them?

I-Thou-It relations

At the beginning of my project, most of the teachers had agreed to participate because they were interested in helping me with my research and they anticipated that they might learn something about mathematics from the experience. In the opening interview, Denise hoped to increase her skills and comfort level with mathematics and liked "the idea of having the fresh new ideas from the university or from you because I think it's easy to get stagnant." Christine wanted to increase her confidence in her ability to teach mathematics. She was interested in exploring strong connections among subjects across the school curricula. Both Denise and Christine liked the idea of working with other colleagues. Martha desired to "understand more about how the kids understand math" and suggested that she needed to improve her ability to teach mathematics so that she could increase "the number of paths to get there. " Likewise, Barb wanted to learn more about the history of mathematics "so it will give me another avenue to help [the students], to help teach another way. The more ways I have, the more I can reach those students that need that." All teachers viewed their participation in the project as professional development.

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[28] See Chapter 8 for the first two conclusions of the study.

During the concluding interview, I asked the teachers to reflect on what they had learned. All teachers claimed that their expectations for their own professional development had been met, indeed exceeded. Three teachers mentioned that my work in their classrooms (see Interlude 1) was integral to the positive experience. Christine's interest in pursuing further study and her clarity of the value of this experience was augmented by my participation as a teacher in her classroom. Because of what she had observed as I taught her students, she wanted to learn more about the early mathematical developments in other countries:

> Some of the games that you did with the kids were from all over the world and could easily be knit into something that's like an anthology, something where you put together folktales and math combined.[…] I can see some value with something like that. From both sides. From the history side, from the math side, from the language side."

Christine was able to envision a different approach to teaching mathematics in part because of the work I was doing with her students. She connected my time in her classroom to her own professional learning: "It's [professional development] but it has direct effect for the kids."

When asked what she thought was important, Barb commented that she appreciated seeing how I was introducing the history of mathematics to her students. She was interested in her students' process of working mathematically on the tasks I presented:

> You had all those wonderful problems that you were working at with the children. How do you get them to discuss the way in which they solved their problems? How do you get children to do that, to learn from their process?

Denise said that what made this experience worthwhile for her was my enthusiasm for the topic: "It has to stem first from your love, right? Because what you are interested in or what you do really well at is what you're going to teach well." She implied that my participation in her classroom was instrumental in encouraging her to consider something new in mathematics.

The teachers seemed aware of the paradigmatic nature of the mathematical tasks I was engaging their students in during my work in the classrooms. Christine remarked:

> My kids would come to me at the end of that time or at lunch or whatever it was, and go, "You know what we were doing?" And then they'd start going through the whole story and then they'd bring me their paper and they'd go, "And this is how we solved it. And this is…". And they would retell what they [had done], almost like they were reliving what they were doing. So I got a lot of feedback from them each time.

It was not until the final interviews that I realized how important my classroom participation had been to the teachers' mathematical development. They were able to observe my implementation of historical stories and appreciated the effect of this on their students (see Interlude 1). We were able to engage in weekly conversations that extended the ideas with which we had grappled during the group meetings. They were able to share pedagogical insights with me during my classroom visits. In particular, Christine described her ability to integrate children's literature into her mathematics lessons. Denise commented on her use of journal writing with her students. Martha came to understand that she was more suited to teaching in a middle-school environment and applied for a transfer to a middle school. She was excited about the possibilities of teaching mathematics through story and integrating it into her language arts, social studies, and health curricula. Barb wrote an article for the provincial teachers' mathematics journal explicating what she was learning about her assessment practices. Perhaps the teachers' learning could have been a more explicit part of our group conversations throughout the project. Certainly, my presence in the classrooms seemed to contribute to robust relations among *I*, *Thou*, and *It*.

Thou-Thou-It **relations**

Another significant part of the professional learning experience for these teachers was the development of relations among group members. The relationships were established and strengthened during the group conversations and classroom visits. Part of my decision to provide release time involved my exclusion from the professional development setting during the implementation stage of the research where teachers were visiting one another's classrooms. Thus, I created situations where the teachers were participating in the professional development task without me. For me, there was a tension as the *I* was removed. The teacher observing the lesson implementation became the researcher, thus replacing the *I* with another *Thou*. I had no first-hand knowledge of what happened during this stage but had to rely solely on their reports. In this part of the professional development task, these *Thou-Thou* relations were linked to the teachers' coming to know mathematics.

Barb highlighted the importance of the group interactions: "All of the discussions that we've had together as a group have really got me looking at math in a different way." Martha identified the group conversations as necessary to her own learning about the history of mathematics. When asked what was impacting, she said:

> I just think some of the discussion that we had about zero and where it comes from and the different ways that it's developed. Like counting something on body parts and the different bases that different civilizations used for math.

Christine's view of mathematics was challenged through the conversations as she realized that other teachers struggled with teaching mathematics. When asked what had influenced her the most, she responded:

> I still think it's just the opportunity to talk with others and recognize that [our] common experience is one of not understanding things that seem to be so simple. I think that's still what it is. And then gathering an understanding from others that there are options of ways you could handle this, that they went through the same thing for a good length of time and through their teaching experience before they were able to make some connections for themselves, too.

Her ingrained view of being unable to do mathematics shifted as she recognized that teaching mathematics was complex even for those who were adept at doing mathematics. Rather then commiserating with each other about the difficulty of teaching and learning mathematics, the teachers sought to share positive experiences of teaching the specific mathematical concepts and Christine appreciated learning from their experience. She commented, "I learned that I could handle math better than I thought I could" and concluded that she had some ability to continue to learn how to do mathematics. Similarly, Martha was impressed by "the collaboration and just seeing that other people have some of the same struggles that I do. Because that isn't something we get a chance to talk to each other about."

For these teachers, the value of sharing craft knowledge among themselves was considerable. There was a clear focus on mathematics learning and pedagogy as collaboration with colleagues was emphasized. All teachers appreciated learning from other teachers. Martha suggested that "the kind of [professional development] that we did in observing and being able to talk to people in meaningful conversations" was a worthwhile form of professional development. For Barb, it was such a positive learning experience because "we've learned so much from each other." Denise commented:

> We're telling people all the time we just don't get to see other people's programs and other people's classrooms enough. We're in our own little [world]. We're in the school with all these teachers and all these fabulous ideas and resourcefulness and we go into our classroom and close the door. And it's sad. Because what a wealth of information and knowledge that's out there. And so that idea of coming together and meeting as a group was just the best kind of PD that you could have. Really. That's what we want as teachers and that's not what they're giving us. The chance to meet and collaborate and talk.

The group conversations were perceived as non-intimidating. Denise said, "You just have a chance to express your thoughts from the readings. It's just been great and really fun to get to know everybody better." The conversations were viewed as personally challenging because they stimulated ongoing learning about ideas not previously considered.

While group discussions were identified as an important aspect of the professional development setting, relations of *Thou-Thou* were further developed through the classroom visits. Martha said, "I think one of the most powerful things that came out of this for me was the chance to go into their classrooms." During the implementation of the lesson, she appreciated "seeing how the four of us each took and did something different with it" and was fascinated by the different approaches.

Connections made between teachers and schools were viewed by Christine as promoting strong, long-term relationships. She appreciated seeing how other teachers taught and remarked that very few people got the opportunity to participate in such a meaningful approach to professional development. "Even just that alone" was viewed as worthwhile.

All of the teachers noted the significance of being able to listen to and observe the students who were engaged in the lesson. Denise suggested that when she was teaching, she was unable to listen to the students but as an observer she had the opportunity to walk around and interact with the students as they were doing mathematics. She viewed this as a very enjoyable experience. Martha remarked that all teachers should get a chance to "sit down and talk to the kids or help them with something that they were stuck on or talk to other staff" and concluded that "we need to find a way to make that happen more often."

An unanticipated consequence of working in the classrooms involved my relationships with the students. Building strong *I-Thou* relationships with them helped ease the workload when the teachers left their classroom to visit the other teachers.

Martha found that she was not pressured and did not worry about her students because "I knew the kids knew you and you knew them and I knew that things were going to be okay here. And I don't always feel that when I go off and leave a sub here."

The intertwining of relations among *I*, *Thou*, and *It* was evident in the responses given by the teachers to my questions regarding the effect of this experience. My participation in the classrooms helped establish strong *I-Thou* relations with the teachers as I worked intensely alongside them and prompted ongoing conversations about mathematics as we worked on the history of mathematics within our group meetings. The strong connection to classroom practice was seen as valuable and encouraged strong *I-Thou* relations as I was viewed as part of the professional development setting.

Our interactions seemed qualitatively different from Buber's notion of *I-It* relations. Barb remarked that we all had come together with an open attitude and that "no one ever felt that they were perhaps better than the other." A willingness to engage in mutual dialogue was evident. My participation in the project was perceived by some as representative of good relations between teachers in the field and teacher educators at the university. Martha commented:

> Help us find a way to do more of that kind of [professional development] and come at it from the ivory tower. I remember you saying at the beginning that this was something that you wanted to be meaningful for us in the classroom and it has been that. And that doesn't always happen at the university level. It hasn't in my experience anyway.

At the beginning of the project, I indicated that I sought a participatory encounter with the teachers 37). How was this enacted within this professional development setting? How did each participant contribute toward establishing strong I-Thou-It relations?

Robust relations among *I*, *Thou*, and *It*

Each participant contributed to our dialogic community in a variety of ways. In this section I focus on the teachers' perceptions of their roles within the group. My own struggle with this was presented in Interlude 2.

It was obvious from the beginning of this project that there was much diversity within our group. In this chapter's opening conversation, Barb noted diversity as the number one reason for her positive experience. Later in the interview, she said, "Each one of us brings something different to that table. Even though we're all teaching different age levels, we're pretty close." Denise viewed diversity as a strength of the project as well:

> I think being from different schools really helped. Just bringing different life experiences, different ages. We're all different ages, different stages in our life with Martha with little kids and Barb with grown up kids, everybody with her own knowledge. It was just a nice mix of people. I really enjoyed meeting everybody and working along with them.

Personal experiences were intertwined with professional stories as we engaged in the group conversations. Different life experiences and professional backgrounds added to the richness of the setting.

The willingness to share personal struggles and triumphs was appreciated by the teachers. Martha commented that she could "go into a place that felt safe," where "nobody was judging anybody." The acceptance within the group was evident to Barb as well, especially when listening to stories of struggle:

> Although it was a diverse group, everyone was very honest which I really liked. Christine was honest about [the fact that] she was afraid of math. She hated it. And so that was really [important]. She was very upfront with us and I think that was wonderful. We all respected that.

During the project, Barb emerged as a leader. She considered herself a mathematician, yet acknowledged that "I still have lots to learn." The mutual respect she exhibited and encountered allowed her to express her love for mathematics and to ask provoking questions to prompt conversation:

> I feel that with the years of experience that I have, it's time to also pass some of that stuff down. And maybe not just saying, "I found this was the best way" but saying, "Have you thought of [this]?" or "How do you handle this?"

Christine was greatly affected by Barb's expertise. Although Christine did not verbally contribute to our conversations a great deal, she described herself as a sponge who was "entering in" through listening:

> [I was] trying to get a hold of what people were saying and what we were looking at and what we were trying to discuss. That was primarily my entering in at this point. I think it would be more participatory in the future as time goes. Because my footing's a little sure as time goes.

She suggested that one way she participated in the group was by hosting the meetings at her school. It was evident that she felt accepted by the members of the group and felt free to participate according to her own comfort level. She suggested that she benefited from what she was hearing and seeing around her and risked teaching mathematics in front of her colleagues during the classroom visits. Her confidence in teaching mathematics increased because she was supported by the teachers during the visitations.

Martha had a long history of involvement in professional development initiatives. Using her expertise in this area, she reminded us of the strengths of this approach:

> It was very informal. There wasn't a report that I had to write and prove that I had learned something. I was treated like a professional. Go and watch this other professional do what they do and get out of it what you will.

Martha also noted that unlike other professional development contexts where the participants are strangers, she liked that she could visit the classroom of a friend. Interestingly, Martha did not know any of the teachers prior to beginning this project; they had all become friends by being part of this collegial community.

Denise believed that participation by all of the teachers in our project was "meant to be." She appreciated how each teacher "was willing to back down or to share or to join in." She suggested that nobody had "jealous feelings" or the need for "attention-seeking" or "wanting the glory." During one of our group meetings, we talked about whose names would be published as authors of the book. Although Denise had written the original story, the other teachers had worked intensely with it while teaching the lessons to their students. I felt that we needed to decide as a group how we would view possible authorship and copyright issues. Our conversation became a bit tense as the teachers ceased to talk. The awkwardness of the situation was evident. I suggested that Denise might retain copyright of the book while all of the teachers would be included as authors of the corresponding teaching resource we hoped to publish. The teachers responded overwhelmingly in favour of this suggestion. During the final interview, Denise commented on this particular conversation by saying that the teachers "were more than gracious. I mean, they didn't have to do that. And I wouldn't have suggested otherwise."

The robust relations within the group enabled the teachers to support each other in their professional growth. This interaction continued outside of the group meetings and classroom visits as teachers became friends. Drawing on such conversations with the other teachers, Denise said:

> I've just really enjoyed the mutual idea of all of us coming together for the same thing. Not really knowing, I guess, what I was getting into when we started and thinking just how [...] we don't want to stop. This has been really good. More than met our expectations.

An unwillingness to stop participating in the project seemed to indicate that the teachers were finding the experience worthwhile. They wanted to extend the study. Clearly, teacher learning occurred in this professional development setting because relations among *I*, *Thou*, and *It* were robust.

When I consider the structure and the content of this particular professional development setting in relation to the comments made by the teachers participating in the study, I wonder how the identified strengths and shortfalls of the professional development design can be addressed. How can the study of the history of mathematics become a more significant part of the teachers' experience? How can components of my participation in their classrooms, group conversations, and classroom visits be expanded?

Gazing Ahead

In Chapter 2, I investigated a variety of approaches to professional development in elementary mathematics education. Among these was the use of teacher study groups and I was particularly influenced by this research when designing my own study. I was drawn to the promotion of teacher autonomy and the emphasis on teacher-initiated collaboration. This approach to professional development seems to enhance the aspects of my study that the teachers felt were most significant and appears to address my concern about the fragile nature of attending to the mathematics through the study of its history. In this section, I investigate study groups as a possibility for encouraging strong relations among *I*, *Thou*, and *It* and consider the methodological aspects of my study that I would change.

The study of the history of mathematics within this project was tenuous. The mathematization of the story by the teachers could have been enhanced by engaging in a more in-depth study of the mathematics at the beginning of the project. For example, I could have been more persistent in offering tasks involving ancient number systems and we could have read a variety of historical accounts on the origins of numbers rather than relying on one novel. Although the teachers resisted the paradigmatic dimensions of mathematics when implementing their lesson, perhaps this would not have been an issue if we had focused on the study of the history of mathematics before we began writing the story. Maybe we could have worked for a longer period of time planning the student tasks of the lesson and investigating the historical problems inherent in the topic.

An increased time-allotment to the lesson planning within this study might have prompted more research-based conversation within our group. After identifying a topic and before planning the lesson, teachers could have examined relevant teaching and learning materials, investigated and assessed a variety of teaching methods, researched the process of student learning, and explored the educational literature on the topic. The focus on research may offer to enhance the teachers' paradigmatic encounters with the mathematics.

The presumed ease of co-construction as reported in the literature (see Chapter 2) was problematic for me. Throughout my study, I grappled with my role as teacher educator within the group. I was aware of both my need to be seen as a teacher and the teachers' resistance to view me in this way (see Interlude 2). I was reluctant to demonstrate my mathematical expertise because I did not want to disrupt the developing relations of *I-Thou* or *Thou-It*. However, my choice to do this may have limited the depth of study. I could have, and perhaps should have, participated more actively as a voice of mathematics and its history.

It is clear to me that teacher educators with expertise in mathematics education have a unique role to play, especially when working in an elementary setting. A teacher educator's role could include offering an idea for the initial topic, demonstrating responsiveness, flexibility, and expertise in offering mathematical tasks for teacher learning, administering the study, and engaging teachers in conversations about theory of mathematics pedagogy as an outcome of their observations.

It is also apparent to me that academic researchers fulfill a necessary role in the group if the goal of the study group is to increase student learning and create teacher-centred theories. In the absence of experienced teacher leaders in elementary schools, administrators often get involved. This can be problematic if the agenda of the study becomes imposed as evident by the change in teachers' autonomy within professional learning communities that have become mandated by the government in this particular province. When participation becomes compulsory, relations of *I-Thou-It* can become strained. If researchers were to initiate the study groups, teachers' agendas would not necessarily be tied to district and school agendas. Strong *I-Thou-It* relations between the academy and the field could be fostered and theories of education could be robustly linked with classroom examples. Differentiated expertise within the group could be embraced.

Robust *I-Thou* relations were enhanced by the collaborative setting. Participants were respectful of the differentiated expertise evident within the group. My work in the classrooms and the teachers' participation in classroom visits strengthened *I-Thou* relations. Robust *Thou-It* relations were encouraged through participation in group discussions and observations of the classroom implementation of the story. These could have been further developed by my own participation in the classroom visits and by a more thorough explication of our individual professional learning through shared reflections. A more in-depth study of the history of mathematics from both a paradigmatic and narrative perspective could have been engaged in if we had allocated more time for this. The teachers' desire for ongoing relations represents a limitation of the design of this particular study. Continuing the project for another year may have offered expanded possibilities for *Thou-It* relations. The potential for creating a series of historical stories and related student tasks is inspiring. Robust *I-It* relations were prompted by my new understanding of the role of differentiated expertise within a professional development setting. My own confidence as a researcher, as a teacher educator, and as a mathematician has increased as I have come to understand the value of what I bring to the setting (I explicate this further in the Epilogue).

The proposed methodological changes presented above indicate that I have learned much about fostering and sustaining a dialogic community within an elementary mathematics professional development setting. What are the implications of my study when considering preservice teacher education?

Reflecting on the conclusions of this study, I am aware that fostering conversations about mathematics as a humanity can be done effectively in the context of the history of mathematics. Incorporating the stories and lessons created by the inservice teachers within the preservice program would provide an opportunity for initiating these conversations. As a teacher educator, I am motivated to create tasks for these students that introduce the classic stories and problems of mathematics. Students could be encouraged to engage in assignments involving study projects where they locate and implement historical stories within their practicum settings. Much like my own work with the students of the teachers in this study, I need to get to know this *It* of the preservice setting. In addition, I would like to research conditions of synergy within the preservice context and investigate how inservice and preservice education can be integrated. Thus, this new *It* becomes the focus of the next stage of my research into teacher learning.

As I write this conclusion, I am aware that my story is incomplete. It is difficult to express in words the impact of this research process on my understandings of self. The tensions reveal my struggle to evolve from a classroom teacher to a teacher educator. The process of writing this dissertation has made explicit the discomfort, joy, novelty, and grief I have experienced throughout this time. The themes throughout my writing are about possibilities for relations within an inservice group: relations among mathematics, the participants, and myself as a teacher of teachers. I look forward to further encounters within dialogic communities of preservice and inservice teachers as I participate fully as a teacher educator and researcher of mathematics education.

Epilogue – Looking Back

Mutual Learning and Encountering *It*

> *In order that [a person] may coherently further the liberation and actualization of that unity in a new accord of the person with the world, [...] the educator must stand again and again not merely at [one's] own pole in the bipolar relation, but also with the strength of the present realisation at the other pole, and experience the effect of [one's] own action. [...] Educating is only possible to the one who lives over against the other, and yet is detached. [...] Every I-Thou relationship, within a relation which is specified as a purposive working of one part upon the other, persists in virtue of a mutuality which is forbidden to be full.*
>
> *(Buber, 1958, pp. 122-123)*

Buber believed that in educational settings, *I-Thou* relations between the genuine educator and the student are by their nature not fully mutual. He described educational relationships as bipolar: teachers and students are at extreme opposites within the relations. Teachers must be detached because they are focused on affirming the student. They act on the student and then assess the effect of their actions. The student is not concerned with living "the teachers' part in the common situation" (p. 122). According to Buber, the denial of full mutuality within educative relations is necessary.

Looking back at my recollections explicated in the Prologue, I am aware that this quality of relationship existed within many of the professional development settings I experienced. Traditional approaches to professional learning involved a one-way relationship from *I* to *Thou*. *Thou* acting on *I* was neither anticipated nor expected.

As a researcher attempting to make theoretical sense of my data, I believe that the type of educative relationship proposed by Buber is inappropriate for professional development settings. Reciprocity is necessary if professional learning is to occur in an authentic way. Researchers entering relations of *I-Thou* with their participants have the unique opportunity to learn. For me, this realization occurred in the final stages of writing this dissertation. Through the research process, I have become convinced that learning within this particular setting has become a mutual encounter. I challenge Buber's notion that *I-Thou* relations within educative settings are necessarily detached. I do not believe it has to be this way.

In the Prologue, I explicated one context where synergy occurred. Reframing this notion with respect to relations, a synergetic encounter can occur when full mutuality is realized. In this particular study, synergetic relations did not evolve until the classroom visits took place. They were hampered initially because I did not enter with my whole self. When I experienced tension with my teaching self, I could not enter into a genuine meeting with the teachers. I was not one of them and I did not acknowledge my experiences of *Thou* acting on *I*.

I did not enter the study with my whole self as a researcher. Throughout the initial group meetings, it was difficult to work on *I-It* while working on *I-Thou*. Buber suggests the opposite: "The improvement of the ability to experience and use generally involves a decrease in [a person's] power to relate" (p. 92). It is possible that this occurs in certain situations, but I did not experience it. Rather, my intense work on *I-Thou* relations inhibited my relations with *It*. My relationships with the *It* of mathematics and the *It* of the professional development task were not evident at the beginning because of my ambivalent attitude toward my role as a person with mathematics and research expertise.

Throughout this study I grappled with how mathematics could be viewed as a humanity. The creation of a parable fostered conversations about the philosophy of mathematics, an area of thought often omitted in most school experiences. Teachers' disembodied images of mathematics as a collection of disjointed facts were challenged by the integration of mathematics into a lifelike, narrative setting. However, seeking a middle ground between paradigmatic and narrative dimensions of mathematics proved difficult because of the teachers' tendency to attend to what they were comfortable with.

At the beginning of the study I was interested in how the teachers' experiences of mathematics, their views of mathematics, and their content knowledge of mathematics could be altered. While I believe the teachers encountered mathematics in a new way and I have evidence that their images of mathematics changed, I am challenged by their limited ease with the paradigmatic dimension of mathematics. Perhaps emphasizing the metaphor *mathematics is story* constrained the possibilities for viewing mathematics as both narrative and paradigmatic. I have come to understand that my role might be to provoke some discomfort with the narrative aspects of mathematics within the dialogical community. I believe that my persistent presentation of relevant mathematical tasks to enhance teacher learning will not prompt relations of distance if I am encountering relations of *I-Thou*.

At the end of the study, I was able to envision myself as a researcher and teacher educator. My experience as a teacher provides me with expertise in teaching, but I also now realize that I have expertise in these other areas. I am confident I can now enter into meetings with my whole self and that my evolving self can find a place within a professional development community as a researcher and as a mathematics teacher of teachers.

Writing in Relation

Another area I was interested in was how relations of *I-Thou* could be sustained. Buber suggests, "The individual You *must* become an It when the event of relation has run its course. The individual It *can* become a You by entering into the event of relation" (p. 84, italics in original). When is a relational event finished? Can a relational event continue in the absence of the *Thou*? As a researcher, how can this relational event be sustained?

At times during the writing of this dissertation, I experienced the relational transition of the *Thou* to an *It*. This became especially apparent when I was interpreting our conversations. There was a tension between the *I-It* perspective and my inclusion of myself in "our" group meetings. As I was writing, I grappled with a methodological issue when considering if "I" was a "we". I chose to represent my interpretation of the data using the "I" voice and removed myself from the teachers when considering their relationship to *It*. In this way, the teachers were objectified in a sense because I was writing about them as objects. Yet, I became quite attached to their characters and even accidentally called Christine by her pseudonym when talking with her during the final week. I grew to know them better as I continued to question the data and was fully conscious of their presence when I was writing. I made deliberate decisions about the words and ideas I was presenting. I wanted to acknowledge the *Thou-ness* of our interactions and while their interpretations of the conversations might be different and my interpretation might be thought-provoking, I believed that my writing would not sever of our relationships but open new conversations.

The relational event did not end as planned on December 2. Although it was transformed because I was no longer providing release time, I was presented with an opportunity to begin writing my dissertation while continuing the group conversations. Thus, I was able to ask the teachers about ideas I needed clarification on and offer tentative interpretations for their feedback. I experienced the effect of my actions and my research on the teachers I was working with. Writing in relation helped sustain *I-Thou* relations. When absent, I was not experiencing distance.

On June 28, the last day of the study, I visited Christine's school once again. This time there were no watershed tears. I was recognized by her students and greeted warmly. I remembered their names and they remembered mine. Christine showed me a mathematical project done by members of her class. Her students had written their own interpretation of *Math curse* (Scieszka, 1995) and on the first page was written: "On Wednesday, Mrs. Sterenberg said, 'Everything is connected to math.'" Christine told me that I had made several guest appearances in the student version because "they had to get me in there somehow." I was part of their story. I was known by these students. We met each other once again.

During my conversation with Christine, I was reminded of the importance of *I-Thou-It* relations and my original idea of how stories could situate mathematics within a human context. Reflecting on the past year, Christine said:

> You just don't realize what an impact [using stories in math class] can have. That is well worth it. Everybody wants to have an impact in the classroom. Everybody wants the kids to go home with memorable things that they take with them from one year to the next. And if you can be memorable with something as simple as telling stories that make knowledge connect, then why not [incorporate them]? That is exciting. That makes your job exciting.

I, too, wanted to have an impact on the teachers within this professional development setting. I wanted these teachers to leave with "memorable things that they could take with them" into the next year. I believe that the stories I have told within this dissertation are evidence that they have made *I-Thou-It* connections. Through my conversations with each teacher during the past several months, I know that this has been a memorable encounter for them. These memories help sustain *I-Thou* relations.

While I grieve the loss of part of myself, I have been able to create a new identity through this research process. This has been a transforming experience for me and I am forever changed by our meetings. I know that I can participate in a dialogical, mutual learning community with my whole being. I am exceedingly grateful to the teachers I have had the pleasure of learning with. I am ready for my next encounter.

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Appendix A – Fishing for Participants, Swimming in Schools

Challenges faced by researchers when engaged in qualitative research abound in the literature. These range from dynamics of race, class, and gender to the subjectivity of the participant-researcher relationship. Although research studies investigate these issues, few discuss the initial challenge of finding participants. In fact, in many studies it is evident that the researcher has an established relationship with a group of people from whom participants are selected. What happens when the researcher is not an insider? This paper describes the journey of one such researcher and considers the viability of engaging in qualitative research.

Watering down my Enthusiasm

The lake is like glass, a reflective mirror of peace and calmness. Purity is an essence of this transparent, colourless, tasteless, and inodorous water. It is tranquil but alive with possibilities.

I began my research project with hope and excitement. I had passed my candidacy exam, I had received ethics approval, and I had time to devote to my study. However, having relocated to an unfamiliar city, I had left behind my friends and colleagues in the field. My proposal stated, "Within a common geographical region, three participants interested in teaching mathematics through historical stories will be invited to participate in my study." In my naivety I believed that I would be able to find the three necessary participants for my research by contacting teachers directly. Herein rose the first surging wave.

Going Fishing

Plans to develop a relationship with a number of teachers evaporated as I was informed of the protocol for contacting potential research participants. I was required to apply in writing to the division office where the superintendent would grant school access. After I had received permission, I would contact principals of potential sites and explain my study to each of them. If they agreed that my study was worthwhile for their staff members, they would distribute a letter I had written to individual teachers inviting them to participate in my research. Then I would wait for responses.

Of the six school districts that I contacted, I received a negative response from one, no responses from 3, and a positive response from two. Because I was interested in working with teachers in a common region, I chose one of the districts to pursue. Of the 11 principals that I spoke to, I was granted 7 short meetings ranging from 5 – 30 minutes. In this limited time I had to make a good impression as I communicated the complexity of my research study and established an initial relationship with the administrator in order to move to the next stage. If the meeting went well, the principal would agree to distribute letters to her/his teachers. Many principals offered to talk to their staff members personally, others simply placed my letter in the teachers' mailboxes. The research process was muddied by a quantitative dimension as I become fixated on statistics.

Except in one case (which I discuss later), I did not have the opportunity to meet individual teachers personally. This meant that potential participants would have to dive in without knowing me. During the first four months, two teachers responded. Rather than searching for like-minded, willing participants, I was fishing for anyone who would bite. Shifting from merely being a fish out of water, I became a fisher.

Sinking or Swimming

In order to do my research, I wanted three participants. Somehow I needed to catch them, so I cast my net further and contacted schools in outlying regions of the district. I was becoming desperate. As I talked with principals, I began to emphasize the benefits of the research. The teachers would have an opportunity to work with other teachers in the district, they would experience a sustained opportunity for professional development, and I would offer the possibility of co-authoring historical fiction books and annotated software as resources for teachers. In addition, I would be willing to provide release time for the teachers and would be willing to work with other staff members in their classrooms. Essentially, I was willing to do whatever it took to lure the teachers. These incentives became excessive and I began to wonder if the bait was worth the bite. But I was anxious that I would not be able to proceed with my research.

When one administrator told me that I had hit the jackpot and that he had a teacher who would definitely be interested, I did little to mediate the pressure he exerted on this teacher to become involved. In order to reel this teacher in, I pressed for an interview and she complied. Grappling with ethical issues, I met with her informally and explained that she was not obligated to participate and that I would inform her principal that I was withdrawing from the site. This was a tough decision because I felt that if I pressured her, she would participate. And I desperately needed another participant. However, I realized that this would not help establish a trusting relationship and my data would be contaminated because she was not a willing member. This encounter proved to be a watershed with respect to my attitude as a researcher. I was swimming again.

Encountering Dams

Having the patience and persistence to continue to meet with principals was challenging. One situation was particularly discouraging. During a telephone conversation with a principal, I was offered an opportunity to speak to teachers at a staff meeting the following day. I quickly rearranged three commitments on my schedule to accommodate this possibility. When I arrived at the appointed time the next day, I waited for twenty minutes before the secretary finally apologized on behalf of the tardy principal and offered to introduce me to the teachers. The first teacher was absent. The second teacher did not arrive until later in the afternoon. The third teacher was less than impressed that he had to meet with me. In fact, he stated outright that he did not have time for "you people from the ivory tower." I explained that I was a teacher and wondered if he could spare five minutes of his time. He consented but continued to mark student assignments while I endeavoured to lessen his antagonism and spark his interest in my research. After exactly five minutes, I was ushered out of his classroom. This experience underscored the importance of developing a relationship of trust with participants. Perceptions of the researcher as an external authority proved hard to alter in this case. Previously damaging connections with the research institution seemed to prevent this teacher from listening to my proposal. I left questioning the professionalism of some teachers, principals, and researchers.

Recovering slightly, I decided to contact one of the teachers who had been absent. After listening to my description of the project, she decided that she would like to participate. Initially I was quite pleased. Now I had three participants. However, it soon became clear that this teacher had her own agenda for the research. Our conversations became imbued with subtle conditions for participation. For example, at one point she suggested that she would meet with me if I would attend a concert she was organising. In addition, she proposed changes to the study, replacing the creation of historical mathematical stories with the scripting of a dramatic presentation. I suggested to her that perhaps this research project was not a suitable match to her own agenda. Again I was disappointed in having to turn away a seemingly willing participant. Two participants remained.

What followed several days later were the dreaded words: "I hate to let you down." Situational factors contributed to one of the participant's decision to opt out of the study. Her teaching assignment had changed to an older grade and her subsequent request to teach part-time had been granted. She didn't feel that she could commit to the project and felt it was unfair to both of us if she continued. With deep regret I affirmed her decision. Now I had one participant. That night I reread my research proposal and confirmed my commitment to the study. Although greatly discouraged, I continued to contact principals and arrange meetings. Moving forward, these disappointments became water under the bridge.

Flooding

It has been said that when it rains, it pours. As May approached and the school year drew to a close, teachers began thinking about professional activities for the upcoming year. Within two weeks, I received four telephone calls from teachers who wanted to know more about my research. In order to avoid forced participation and encourage shared intentions, I suggested that I meet with each individual to introduce myself after which time I would record an initial interview if he/she was interested in participating. These meetings lasted for about an hour and all four teachers felt comfortable in participating in the study. Instead of the three participants I had wanted, I now had five enthusiastic and willing teachers to work with. We decided to have one group meeting before school ended so that we would be ready to begin in September.

Building Bridges

With the task of selecting individual participants completed, I was still concerned that we develop trusting relationships as a collective. Five participants would provide a number small enough to encourage active engagement and large enough to ensure diversity. My conversations with each teacher suggested that there was a range of teacher experience, expertise, and interests. I hoped our group of participants will organize itself and, as such, I wanted to be careful not to assume an expert role of manager. My role as a researcher would be one of a participant, dispersing control as our common project unfolded and shared insights emerged. I wanted to be part of the collective in my role as a teacher-researcher.

What became evident at our group meeting was that these teachers were genuinely interested in their professional development. They were willing to take risks: risks of revealing their inadequate mathematical knowledge, risks of showing their discomfort with constructivist teaching methods, risks of wasting valuable time engaged in a new research project, risks of having strangers evade their classroom space. As the conversation progressed, they began referring to common acquaintances. The connections they identified resulted in much laughter and surprise. One participant's sister was a good friend of another participant. Several participants had taught at the same school at different times and were connected through the staff members still teaching there. Other participants had worked for the same principal at different schools. These bridges seemed to establish common understandings of place and context.

A common interest in historical mathematical narratives emerged as the basis for returning the conversation to the research project. A number of questions were posed and we talked about ethical issues and confidentiality. I wanted the teachers to feel assured that this would be an empathetic endeavour. While discussing possible activities, one participant told me that I was the expert. It was clear that they were expecting some direction and that the open-ended nature of the timeline was disconcerting. I decided to share my vision of how this might look and this seemed to inspire more confidence in the project. I stressed that I was flexible but at this time it appeared that my role needed to be negotiated as the project unfolded. For now they were depending on me to provide initial direction.

As our conversation ended, a few participants remained in the parking lot as they continued their discussion. Even in the busyness of preparing final report cards and completing year-end activities, these teachers were looking forward to participating in the project. I felt a deep sense of privilege to be working with such dedicated professionals.

Taking to the Water

The process of finding participants immersed me in the practicalities of engaging in qualitative research. Ethical considerations surfaced throughout the six months. Questions from the ethics application begged further deliberation. How would I provide opportunities for potential participants to exercise their right to not participate? How would I provide opportunities for my participants to exercise the right to opt out without penalty, harm, or loss of promised benefit? How would I address privacy, anonymity, and confidentiality issues? Since my study depended on strong relationships with my participants, the importance of ethical living was paramount. Time was required to build these relationships. Certainly I came to understand the importance of ethical responsibility as I interacted with teachers who might become participants in future research studies. Moreover, leaving a good impression would occasion good researcher-participant relationships for those who might follow me.

Closely connected to ethical considerations were contextual issues. Requesting participants during report card time limited positive responses. Planning for research to begin in spring was viewed as impossible by most teachers. Pressure by administrators compromised the ability for participants to give free consent. The school context impacted my search for willing participants.

Perhaps the most surprising aspect of finding participants was my own personal stake in the process. I felt I was applying for jobs and meetings with principals had a flavour of a job interview. I returned many times to my proposal as I reconsidered the importance of my research. The process of finding participants provided me with many opportunities to reflect on my role as a researcher.

Working in relation requires ethical responsibility, adaptation to the context, and the willingness to become more self- conscious. The challenges inherent in qualitative research can be frustrating but the benefits of becoming an insider by engaging in qualitative research far outweigh the obstacles. Engaging in qualitative research is not only viable, but also inspiring as the unexpected is embraced.

The lake ripples, altering the clear reflection. This water is not pure, colourless, tasteless, inodorous; it is vibrant, unpredictable, effervescent. It is teaming with verve. Finding my place in this research school, I strive to become a synchronized swimmer.

Appendix B – Timeline of Meetings

May 4-27, 2004 August 26, 2004	Initial individual interviews	
June 17, 2004	First group meeting	Participant introduction; overview of project; tentative planning of first meeting
September 8, 2004	Release time (my classroom teaching) begins	
September 9, 2004	Second group meeting	Introduction; metaphors of mathematics; history of numbers readings
September 23, 2004	Third group meeting	History of numbers readings; nature of mathematics and language
October 4, 2004	Fourth group meeting	Investigation of story; teacher resources
October 14, 2004	Fifth group meeting First round of classroom visits	Story writing; mathematics is story; task creation
November 4, 2004	Sixth group meeting Second round of classroom visits	Lesson plan; feedback on classroom visits
November 18, 2004	Seventh group meeting Third round of classroom visits	Lesson implementation; feedback on classroom visits; selection of lesson artifacts for CD resource
December 2, 2004	Eighth group meeting	Implementation feedback; CD resources
January 13, 2005	Ninth group meeting	Agenda for future meetings; preliminary illustrations
January 19-27, 2005	Individual interviews	
January 27, 2005	Tenth group meeting	CD resources
February 3, 2005	Eleventh group meeting	CD resources
March 3, 2005	Twelfth group meeting	CD resources
April 7, 2005	Thirteenth group meeting	Meeting with illustrator
May 18, 2005	Fourteenth group meeting	Celebration supper
June 21-28, 2005	Final individual interviews	

Appendix C – Story Samples

Sums of Series

Carl snatched at the satchel and spun around to say goodbye to his mother as he scrambled for the door. Running to join the other children, he smiled briefly in anticipation of what the school day will bring. He was a mischievous boy who loved to present new approaches to the way things were supposed to be. Today, however, his teacher was in a foul mood and assigned a monstrous task to the students: add all the numbers from one to one hundred. Carl knew the teacher wanted to keep them busy all morning and frowned at the lack of imagination in this assignment. Slowly, he started to write down the numbers but then was distracted by patterns he saw. Instead of adding the numbers one after the other, he rearranged the numbers and reached the answer after just a few seconds. In delight he raised his hand to get the teacher's attention. Alas, the teacher thought he was pretending and scowled at him. Again Carl reached out his hand, this time waving it. The teacher called him to the front of the classroom and scolded him for interrupting the work. As he grabbed Carl's slate, he saw a new way of doing this question. Carl's attention to patterns earned him a free morning.

Johann Carl Friedrich Gauss (1777-1855)

Student Task

Find the sum of the numbers from one to one hundred without using a calculator (e.g. 1 + 2 + 3 + 4 + 5 + 6…). Describe a pattern for this process.

Represent the pattern using shapes.

Allowances

Read: *The King's Chessboard* by David Birch

Student Task

A father complained that his daughter's allowance of $10 per week was too much. The daughter replied, "Okay, Dad, how about this? Will you give me a penny on the first day of the month, two pennies on the second day of the month, four pennies on the third day of the month, eight pennies on the fourth day of month, sixteen pennies on the fifth day of the month, and so on for every day of the month?"

The father quickly calculated that he would pay his daughter only $1.27 the first week, instead of $10, and he readily agreed. Who was the unwise one, the father or the daughter? Show how you know.

Appendix D – Metaphoric Terms

Teaching Mathematics is . . .

Tutoring
Guiding
Lecturing
Remediating
Initiating
Giving voice
Participating
Caring
Fostering
Training
Directing
Modeling
Empowering
Framing
Minding
Educating
Inducting
Edifying
Conditioning
Mentoring
Liberating
Structuring
Listening
Indoctrinating
Informing
Schooling
Mediating
Emancipating
Occasioning
Conversing
Disciplining
Instructing
Facilitating
Improvising
Applying
Connecting
Investigating
Entertaining
Building
Repeating
Translating
Overcoming
Manipulating
Solving

Developing a Metaphor

Mathematics is a . . .
Mathematics is like a . . .

Appendix E – Historical Resources

Aczel, A. D. (1996). *Fermat's last theorem*. New York: Bantam Doubleday Dell Publishers.
Math Concepts: patterns can be used to describe the world and to solve problems; algebraic and graphical models can be used to generalize patterns, make predictions, and solve problems (explicit)
Text Features: non-fiction novel; includes a preface and an index; includes historical explanations and references

Adler, D. A. (1977). Roman numerals. New York: HarperCollins Publishers.
Math Concepts: numbers can be represented in multiple ways (explicit)
Text Features: information text

Cooney, M. P. (Ed.). (1996). *Celebrating women in mathematics and science*. Reston, VA: National Council of Teachers of Mathematics.
Math Concepts: many women have been actively engaged in creating new mathematics (explicit)
Text Features: biographies; includes a table of contents and a preface; ink illustrations

de Guzmán. (2000). *The countingbury tales*. River Edge, NJ: World Scientific Publishing Co.
Math Concepts: games and beauty often compel mathematicians to develop concepts (explicit)
Text Features: information book; includes a table of contents and a bibliography; a historical perspective is given; each chapter differs in degree of difficulty

Ganeri, A. (1999). *Legacies from ancient Greece*. Mankato, MN: Thameside Press.
Math Concepts: mathematics is a cultural activity (implicit)
Text Features: information book; includes contents, a glossary, and an index; is part of a series on ancient civilizations

Garland, T. H. (1990). *Fascinating Fibonaccis: Mystery and magic in numbers.* Palo Alto, CA: Dale Seymour Publications.
Math Concepts: patterns can be used to describe the world and to solve problems (explicit)
Text Features: information book; includes diagrams, a few proofs, and historical notes

Guedj, D. (2000). *The parrot's theorem*. London: Weidenfeld & Nicolson.
Math Concepts: the history of mathematics contains adventures and controversy (explicit)
Text Features: mystery novel; diagrams are included; mathematical developments at various levels of difficulty are presented

Haven, K. (1998). *Marvels of math*. Englewood, CO: Teacher Ideas Press.
Math Concepts: mathematics develops in a social context and is a dynamic cultural activity (explicit)
Text Features: biographies; collection of 16 historical stories; includes a table of contents, an introduction, and an index; included for each story are brief summaries, terms to know, follow-on questions, and activities

Lasky, K. (1994). *The librarian who measured the earth*. Boston, MA: Little, Brown and Company.
Math Concepts: similar triangles may be used to solve problems; angle measurements are linked to the properties of parallel lines (explicit)
Text Features: biography of Eratosthenes; picture book; includes the author's note, an afterword, and a bibliography

Mankiewicz, R. (2000). *The story of mathematics*. Princeton NJ: Princeton University Press.
Math Concepts: human development is connected to the development of mathematical concepts and ideas (explicit)
Text Features: information and biographical text; text is dense and small making it less readable than other texts; includes a foreword, a preface, and an index; includes a high number of illustrations, photographs, reproductions of artifacts and art, and computer-generated images

Mlodinow, L. (2001). *Euclid's window*. New York: Touchstone.
Math Concepts: geometry underlies much of the mathematics developed to date (explicit)
Text Features: biographies; includes an introduction, an epilogue, and an index

Pappas, T. (1989). *The joy of mathematics*. San Carlos, CA: Wide World Publishing/Tetra.
Math Concepts: everyday phenomena can be described using mathematics (explicit)
Text Features: information book; includes historical notes and descriptions of the concept and the mathematician; includes a table of contents, solutions, bibliography, and an index

Pappas, T. (1991). *More joy of mathematics*. San Carlos, CA: Wide World Publishing/Tetra.
Math Concepts: everyday phenomena can be described using mathematics (explicit)
Text Features: information book; includes historical notes and descriptions of the concept and the mathematician; includes a table of contents, solutions, a bibliography, and a cross index with the previous volume

Pappas, T. (1993). *Fractals, googols and other mathematical tales*. San Carlos, CA: Wide World Publishing/Tetra.
Math Concepts: mathematical ideas emerge from human imagination (explicit)
Text Features: historical accounts included; contains a table of contents, solutions, answers, and explanations

Perl, T. (1993). *Women and numbers*. San Carlos, CA: Wide World Publishing/Tetra.

Math Concepts: women are actively engaged in creating new mathematics; numbers can be used to solve problems (explicit)

Text Features: biographies; includes a table of contents, timelines, and solutions to activities; the historical backgrounds of conceptual developments are provided

Reimer, L., & Reimer, W. (1990). *Mathematicians are people, too. Volume 1.* Parsippany, NJ: Dale Seymour Publications.

Math Concepts: mathematics is a human creation (explicit)

Text Features: biographies; includes an introduction for teachers, an introduction for students, a resource list, and a glossary; black and white full-page illustrations

Reimer, L., & Reimer, W. (1995). *Mathematicians are people, too. Volume 2.* Parsippany, NJ: Dale Seymour Publications.

Math Concepts: mathematics is developed and shaped by humans (explicit)

Text Features: biographies; includes an introduction for teachers, an introduction for students, a resource list, and a glossary; black and white full-page illustrations

Schmandt-Besserat, D. (1999). *The history of counting*. New York: Morrow Junior Books.

Math Concepts: numbers were invented; numbers can be represented in many ways; the development of number systems coincided with societal and cultural developments (explicit)

Text Features: historical non-fiction; contains a glossary and an index; colourful pictures

Schwartz, D. M. (2003). *Millions to measure*. New York: HarperCollins Publishers.

Math Concepts: everyday phenomena can be described and compared using measurements (explicit)

Text Features: nonstandard measurement systems described from a historical background; imperial and metric system are presented; detailed illustrations; the center foldout section shows metric measurements; includes the author's notes on the metric system

Seife, C. (2000). *Zero: The biography of a dangerous idea.* New York: Penguin Books.

Math Concepts: zero is linked to infinity; philosophical controversies erupted over the acceptance of zero as a number; numerical data can be analyzed for trends, patterns, and interrelationships; arithmetic operations can be applied to solve problems (explicit)

Text Features: information text; historical ideas and biographies are presented; includes recent developments of quantum zero, black holes, and string theory; contains a table of contents and an index; includes diagrams, drawings, and photographs

Smith, D. E. (2001). *Number stories of long ago.* Reston, VA: National Council of Teachers of Mathematics.

Math Concepts: numbers describe quantities; numbers can be represented in multiple ways (explicit)

Text Features: historical narrative; contains a question box at the end of each chapter, a note to teachers, and mathematical explanations of the puzzles presented; colourful lithographs precede each chapter

Smith, S. (1996). *Agnesi to Zeno: Over 100 vignettes from the history of math*. Berkeley, CA: Key Curriculum Press.
Math Concepts: mathematics has developed over time; mathematics is a cultural activity; patterns and algebraic expressions can be used to solve problems (explicit)
Text Features: non-fiction; includes a preface, suggested uses for the book, an appendix, selected answers, and an index; each vignette is followed by a list of activities and related readings

Swetz, F. J. (1994). *From five fingers to infinity*. Peru, IL: Open Court Publishing Company.
Math Concepts: the history of mathematics is part of the history of humankind (explicit)
Text Features: non-fiction; includes a table of contents, note to the reader, a preface, a bibliography, and an index; is a comprehensive volume

Tahan, M. (1993). *The man who counted*. New York: W. W. Norton & Company. ISBN 0393309347
Math Concepts: throughout history, people have engaged in solving mathematical problems; there are connections between philosophy, religion, and mathematics (explicit)
Text Features: narrative; set in the 13th century on the road to Baghdad; answers are provided within the text; historical references to traditional and classic problems are given

Trumble, K. (2003). *The library of Alexandria*. New York: Clarion Books.
Math Concepts: mathematics develops within a cultural context (implicit)
Text Features: information book; includes a table of contents, maps, family trees, names and terms, a bibliography, suggested reading lists, and an index; full-page colourful and detailed illustrations; includes short biographical notes on Euclid and Archimedes

Vorderman, C. (1999). *How math works*. Pleasantville, New York: The Reader's Digest
Math Concepts: everyday phenomena can be described using mathematics (explicit)
Text Features: activity and information book; historical notes are included; colourful pictures and diagrams; includes a table of contents, a glossary, answers to puzzles, and an index

Wilson, R. (2003). *Four colours suffice*. London: Penguin Books.
Math Concepts: design problems can be explored using properties of networks (explicit)
Text Features: historical non-fiction; includes a table of contents, a preface, notes and references, a chronology of events, a glossary, and an index; contains photographs and diagrams; dense text and high reading level

Appendix F – How Much is Some?

Long, long ago in Ancient Egypt, before there were numbers, things were quite different and much more difficult. People had to communicate their needs using math words such as: a few, lots, more, less, a couple, several, many, a handful, or a bunch.

One day, a hard-working peasant couple named Majed and Neferet took potatoes to the market to trade for goat's milk. They had heard that Wosret, a merchant at the market, was a very fair trader. So Majed and Neferet rushed with their potatoes to make a trade with Wosret.

"We have SOME potatoes to trade, Wosret. Can you trade our potatoes for SOME goat's milk?"

"Yes, I can trade with you," said Wosret, "but, tell me, what does SOME mean?"

Majed looked at Neferet and replied, "We don't know what SOME is, Wosret, but this is what SOME looks like."

Majed then handed a dirty burlap bag to Wosret. Wosret peered into Majed's deep, dark bag and snapped at him by saying, "I can't see SOME. Your bag is too deep and too dark!"

"The potatoes are at the bottom," said Majed.

"I can't see them, barked Wosret, "so you need to show me your SOME."

Neferet stepped forward and helped Majed turn the bag upside down and SOME potatoes dropped to the ground with a thud. "Here is SOME!" Neferet proudly exclaimed.

Wosret threw his head back and laughed loudly while saying, "I traded SOME potatoes with Nubi yesterday and that SOME was much more than this. I cannot trade SOME milk with you today. Come back tomorrow with SOME MORE potatoes."

Neferent and Maject understood what Wosret wanted. Early the next day, Majed and Neferet went to their small field and collected SOME MORE potatoes. Later that day, they returned to the market and went to see Wosret. They gave their SOME MORE potoatoes to Wosret but again Wosret said, "I cannot trade with you today. Your SOME MORE is still not enough. Come back tomorrow with MORE."

The next morning, Maject and Neferet returned to their small field and collected SOME MORE potatoes. In the afternoon, they returned to the market and went to see Wosret. Neferet and Majed turned the bag upside down. The potatoes dropped to the ground with a big thud. Wosret looked at he potatoes; Wosret didn't look pleased. He shook his head and said, "Your MORE and SOME MORE potatoes are still not large enough. I cannot trade SOME milk with you today. Come back tomorrow with SOME MORE potatoes."

Neferet and Majed put the potatoes back into the bag. They walked back through the market and down the hill to their village. They dug up and bagged SOME MORE potatoes. They walked back up the hill to the market and found Wosret. They emptied the bag of potatoes onto the ground for Wosret to see.

Wosret looked at the potatoes on the ground. He was pleased. Finally he was satisfied that the trade could be made. Wosret took the couple's MORE and SOME MORE potatoes and happily handed them a jug of fresh goat's milk in return. The joyful couple headed through the market and down the hill, back to their village.